

Paragraph HW 06
Examples of vector fields
Math 112, Spring 2006

In this homework, you can draw vector fields by hand, if you prefer not to use Maple, but certainly Maple will make your task much easier. Throughout, we refer to the vector field $-y\mathbf{i} + x\mathbf{j}$ as the counterclockwise “swirly”.

1. Find a formula for a 2-dimensional vector field \vec{V} such that the vector at any point in the plane (except $(0,0)$, where \vec{V} is not defined) points towards $(0,0)$ and has unit length. Use algebra to verify that all of the vectors in your vector field have unit length, and draw your example.
2. Find a formula for a 2-dimensional vector field that looks like the counterclockwise “swirly”, except that the length of the vector at a distance $r = \sqrt{x^2 + y^2}$ from the origin is $\frac{1}{r}$. (Note that consequently, it follows that \vec{V} is not defined at $(0,0)$.) Use algebra to verify that the length of the vector at a distance $r = \sqrt{x^2 + y^2}$ from the origin is $\frac{1}{r}$, and draw your example.
3. Find a formula for a 3-dimensional vector field \vec{V} such that the vector at a distance $\rho = \sqrt{x^2 + y^2 + z^2}$ from the origin is $\frac{1}{\rho^2}$. (Note that consequently, it follows that \vec{V} is not defined at $(0,0,0)$.) Use algebra to verify that the vectors in your vector field have the desired length, and draw your example.
4. Find a formula for a 3-dimensional vector field \vec{V} such that for any fixed value of y (i.e., for any slice of \mathbb{R}^3 parallel to the xz -plane), \vec{V} forms a “swirly” centered at the y -axis. Draw your example in a region where the “swirlies” around the y -axis are clearly visible.

Maple: More on plotting vector fields

Code for this assignment. As usual, you can get a copy of the worksheet for this assignment either from the handouts folder, if you’re working in the Math Lab, or from the course website:

<http://www.math.sjsu.edu/~hsu/courses/112/Math112-p06.mw>

Vector fields. In Maple, a vector field in \mathbb{R}^2 is represented by a pair of functions. For example, one standard counterclockwise “swirly” is represented by

```
> [-y,x];
```

In other words, the vector field $-y\mathbf{i} + x\mathbf{j}$ is represented by the matrix $[-y \ x]$. Similarly, a vector field in \mathbb{R}^3 is represented by a matrix of three functions.

Plotting a vector field. To plot the above “swirly” for $-2 \leq x \leq 2$ and $-2 \leq y \leq 2$, use the command:

```
> fieldplot([-y,x],x=-2..2,y=-2..2);
```

Similarly, the radial outward vector field $x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ is plotted by the command:

```
> fieldplot3d([x,y,z],x=-2..2,y=-2..2,z=-2..2,axes=normal);
```