

Paragraph HW 03
Differentiability/Maple update
Math 112, Spring 2006

Let

$$f(x, y) = \begin{cases} \frac{xy^{4/3}}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0), \\ 0 & \text{if } (x, y) = (0, 0); \end{cases}$$

and let

$$g(x, y) = x^{5/7}y^{4/7}.$$

Is f differentiable at $(0, 0)$? Is g ? Both? Neither? More specifically, for each of f and g , if the function is not differentiable, explain how you know that it is not differentiable; and if the function is differentiable, give all the evidence you've found that the function is differentiable. (You do not need to give a formal proof that either function is differentiable.)

Remember that to answer this question, you may wish to consider the following evidence:

1. Continuity of f and g at $(0, 0)$.
2. Partial derivatives of f and g at $(0, 0)$.
3. Directional derivatives of f and g at $(0, 0)$.
4. Tables of values of f and g , zooming in towards $(x, y) = (0, 0)$.
5. Graphs of f and g , zooming in towards $(x, y) = (0, 0)$.

Maple update

Code for this assignment. As with PS02, you can get a copy of the worksheet for this assignment either from the handouts folder, if you're working in the Math Lab, or, as always, from the course website:

<http://www.math.sjsu.edu/~hsu/courses/112/Math112-p03.mw>

Functions with multi-part definitions. In the homework, you have to work with the function

$$f(x, y) = \begin{cases} 0 & \text{if } (x, y) = (0, 0); \\ \frac{xy^{4/3}}{x^2 + y^2} & \text{otherwise.} \end{cases}$$

Since Maple is a full programming language, it turns out that this function can be implemented without too much difficulty. One way to do this is (over):

```

> f := (x,y) ->
> if (x^2 + y^2=0)
>   then 0
>   else x*y^(4/3)/(x^2+y^2)
> fi:

```

Note that if you enter this line by line, you will receive warnings saying something about “premature end of input.” You may safely ignore these as long as you terminate the input correctly with `fi:`. On the other hand, if the warnings really bother you, you can just hit SHIFT-ENTER instead of ENTER at the end of each line except the last one, or you can just type the entire definition on one line.

Graphs of functions of 2 variables in 3-space. Remember that to plot, for instance, $z = x^2 + y^2$, you use the command:

```

> plot3d(x^2+y^2,x=-2..2,y=-2..2,axes=normal);

```

Unlike `plot`, `plot3d` requires you to specifically request axes to appear; if you omit the `axes=normal` part of the command, none will appear. You can also change the type of axes of your plot by using the “Axes” pulldown menu; for instance, you may want to try “framed” axes.

Contour plots of functions of 2 variables. Remember that to do a contour plot of, for instance, $f(x, y) = x^2 + y^2$, you use the command:

```

> contourplot(x^2+y^2,x=-2..2,y=-2..2);

```

This will plot level sets $x^2 + y^2 = C$ for certain carefully selected values of C in the region $-2 \leq x \leq 2$, $-2 \leq y \leq 2$.

Tables of values of a function. The worksheet for PS03 includes a short program for making a table of values of a function. To illustrate by example, the command

```

> deltatable(f,1.0,2.0,0.01,0.03);

```

will produce a table of values of $f(x, y)$ for $x = 0.99, 1.0, \text{ and } 1.01$, and $y = 1.97, 2.0, \text{ and } 2.03$ (i.e., a total of 9 values examined). To look at a different set of values, say for $x = 0.0 \pm 0.01$ and $y = 0.0 \pm 0.01$, you can run `deltatable` using different numbers:

```

> deltatable(f,0.0,0.0,0.01,0.01);

```

One peculiarity of Maple: When working with decimal approximations, which in this class we do most of the time, always enter all numbers, even whole numbers, with a decimal point included. In other words, always use `3.0` instead of `3`.