Sample Exam 1
Math 108, Spring 2016

1. (12 points) Let I be a set, let X be a set, and let $\{A_\alpha \mid \alpha \in I\}$ be an indexed family of subsets of X. Define $\bigcap_{\alpha \in I} A_\alpha$ and $\bigcup_{\alpha \in I} A_\alpha$.

2. (14 points) As usual, people from **TR**acy always tell the **TR**uth, people from **LI**vermore always **LI**e, and we assume that each of A, B, C, and D is either from Tracy or from Livermore. Suppose A and B say:

A: Either I am from Livermore, or B is from Livermore.
B: If C is from Tracy, then D is from Tracy.

Determine whether each of A, B, C, and D is from Tracy or from Livermore.

In questions 3–4, you are given a statement. If the statement is true, you need only write "True", though a justification may earn you partial credit if the correct answer is "False". If the statement is false, write "False", and justify your answer **as specifically as possible**. (Do not just write "T" or "F", as you may not receive any credit; write out the entire word "True" or "False".)

3. (12 points) Let X be a set, and let A and B be subsets of X. It must be the case that if $x \in X$ and $x \notin A \cap B$, then $x \notin A$ and $x \notin B$.

4. (12 points) Let $A = \{1, 2, 3, 4\}$. Then $\{1, 3\}$ is an element of $\mathcal{P}(A)$, the power set of A.

5. (16 points) **OUTLINE** the proof of the following theorem. More precisely, for each half of the proof of the theorem, state the assumption and the conclusion, and work forwards and backwards towards the middle as much as you can by only applying definitions and breaking the proof into cases, as appropriate. **PLEASE DO NOT TRY TO FINISH THE PROOF.**

Theorem. Let A and B be sets. Then $(A \setminus B) \cup B = A \cup B$.

6. (16 points) **PROOF QUESTION.** Let

$$S = \left\{ n \in \mathbf{Z} \mid n = k^2 \text{ for some } k \in \mathbf{Z} \right\},$$
$$T = \left\{ n \in \mathbf{Z} \mid n = m^6 \text{ for some } m \in \mathbf{Z} \right\}.$$

Prove that $T \subseteq S$.

7. (18 points) **PROOF QUESTION.** Let A, B, C, and D be sets. Prove that if $A \subseteq B$ and $C \subseteq D$, then $(A \times C) \subseteq (B \times D)$.

Suggestion: If you get stuck, at least write down the definition of Cartesian product.