

Format and topics
Exam 2, Math 108

General information. Exam 2 will be a timed test of 75 minutes, covering Chapters 10–12 and 14–17 of the Yellow Book and Parts I–III (Sections 1–19) of the proof notes. No books, notes, calculators, etc., are allowed. Most of the exam will rely on understanding the problem sets and the definitions and theorems that lie behind them. If you can do all of the homework, and you know and understand all of the definitions and the statements of all of the theorems we’ve studied, you should be in good shape.

You should not spend time memorizing proofs of theorems from the book, though understanding those proofs does help you understand the theorems. On the other hand, you should definitely spend time memorizing the *statements* of the important theorems in the text.

Types of questions. Exam 2 will feature the same types of questions as Exam 1, including statements of definitions and theorems, proofs, computations, and true/false with justification. Exam 2 may also feature another kind of question that was on Exam 1, namely:

Proof outlines. In these questions, you will be asked to outline the proof of a theorem, and not finish the proof. In such a question, you need to state the assumptions and conclusions of the theorem clearly and split the proof into parts if the proof has an obvious Part 1/Part 2 structure (e.g., if the theorem is an “if and only if” statement). Furthermore, if the conclusion of the theorem involves a “there exists” statement, you should indicate the point where you have to construct or choose a corresponding object (i.e., “Let $p = ?$ ”).

Definitions. The most important definitions we have covered in the Yellow and Blue Book and the proof notes are:

Ch. 10	reflexive relation transitive relation equivalence class	symmetric relation equivalence relation
Ch. 11	partition	block (PS04)
Ch. 12	bounded above upper bound bounded minimum least upper bound greatest lower bound	bounded below lower bound maximum supremum infimum
Ch. 14	function well-defined codomain	map, mapping domain range
Ch. 15	one-to-one injective bijective surjection restriction	onto surjective injection bijection
Ch. 16	composite function identity function	composition inverse
Ch. 17	image	
Proof notes 18	inverse	invertible
Proof notes 19	co-restriction	bi-restriction

Examples. You will also need to be familiar with the key properties of the main examples we have discussed. The most important examples we have seen are:

Ch. 10 Example 10.3.

Ch. 12 Examples 12.2, 12.7.

Ch. 14 Examples 14.5, 14.8.

Ch. 15 Examples 15.2, 15.3, 15.6, 15.7.

Ch. 16 Examples 16.1, 16.2.

You should also be familiar with all of the examples from the Exercises from Ch. 10–12 and 14–17, and you should be familiar with the examples from PS04–06.

Theorems, results, algorithms, axioms. The most important theorems, results, algorithms, and axioms we have covered are listed below. You should understand all of these results, and you should be able to state any theorem clearly and precisely. You don't have to memorize theorems by number or page number; however, you should be able to state a theorem, given a reasonable identification of the theorem (either a name or a vague description).

Ch. 10 Example 10.2.

Ch. 11 Partitions come from equivalence relations, and vice-versa (Thm. 11.4).

Ch. 12 Completeness axiom of \mathbf{R} ; Archimedean Property of \mathbf{R} (either Thm. 12.10 or Cor. 12.11); Well-ordering principle of \mathbf{N} ; Density of the rationals in the reals (Thm. 12.12).

Ch. 15 "Redefinition" of onto (Lem. 15.1).

Ch. 16 Facts about inverses (Thm. 16.4); $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$ (Thm. 16.6); Composition of one-to-one, onto, and converse (Thms. 16.7, 16.8).

Proofs handout, Sect. 18 The inverse theorem.

Ch. 17 Properties of images (Thm. 17.5 and 17.7(1)–(4)).

Other. Please be familiar with the techniques of proof in the proof notes, Sects. 1–19.

Not on exam. (Ch. 14) Characteristic function, step function, greatest integer, floor function. (Ch. 17) inverse image (preimage), Thm. 17.7(5)–(10).

Good luck.