Math 108, problem set 02
Outline due: Wed Feb 08
Completed version due: Mon Feb 15
Last revision due: Mon Mar 21

Exercises (to be done but not turned in): 6.2-6.5, 6.7-6.8, 6.10, 6.12-6.14; 7.3, 7.5.
Problems to be turned in: All numbers refer to problems in the Yellow and Blue Book.

1. Let

$$A = \left\{(v, w, x, y, z) \in \mathbf{R}^5 \mid x = 0\right\},$$
$$B = \left\{(v, w, x, y, z) \in \mathbf{R}^5 \mid y = 0\right\},$$
$$C = \left\{(v, w, 0, 0, z) \mid v, w, z \in \mathbf{R}\right\}.$$

   Prove that $A \cap B = C$.

2. Consider the sets

$$A = \left\{(x, y) \in \mathbf{R}^2 \mid y \geq x^2 + 1\right\},$$
$$B = \left\{(x, y) \in \mathbf{R}^2 \mid y \leq -x^2 - 1\right\},$$
$$C = \left\{(x, y) \in \mathbf{R}^2 \mid |y| \geq 2\,|x|\right\},$$
$$D = A \cup B.$$

   Does $C = D$? Is one of C and D a subset of the other, or is neither a subset of the other? Carefully state and prove your answer. (Suggestion: $x^2 = |x|^2$.)

3. For any integer n, define

$$n\mathbf{Z} = \{a \in \mathbf{Z} \mid a = nk \text{ for some } k \in \mathbf{Z}\}. \tag{1}$$

   Recall that if $a, b \in \mathbf{Z}$, to say that a divides b means that $b = ac$ for some $c \in \mathbf{Z}$. Exactly one of the following two statements is true for all $a, b \in \mathbf{Z}$:

   If a divides b, then $a\mathbf{Z} \subseteq b\mathbf{Z}$.
   If a divides b, then $b\mathbf{Z} \subseteq a\mathbf{Z}$.

   Determine which statement is always true, and prove it.

4. 7.1(b).

5. 7.1(e).

6. 7.5(a).

7. 7.14. (For (a), suppose $(A \setminus B) \cap B \neq \emptyset$, and proceed by contradiction.)