Math 108, problem set 01b
Due: Mon Feb 08
Last revision due: Mon Feb 22

Problems to be turned in:
The goal of this problem set is to outline the proof of the following theorem.

Theorem 1. Let n be a positive integer. If v_1, v_2, v_3, v_4, v_5 are contained in $\mathbf{R}^n$, then

$$\operatorname{span}\{v_1, v_2, v_3\} \subseteq \operatorname{span}\{v_1, v_2, v_3, v_4, v_5\}.$$

Here are all of the definitions you need to make sense of the statement of this theorem.

Definition. For a positive integer n, $\mathbf{R}^n = \{(x_1, \ldots, x_n) \mid x_i \in \mathbf{R} \text{ for } 1 \leq i \leq n\}$.

Definition. For $c \in \mathbf{R}$, $(x_1, \ldots, x_n) \in \mathbf{R}^n$, and $(y_1, \ldots, y_n) \in \mathbf{R}^n$, we define

$$(x_1, \ldots, x_n) + (y_1, \ldots, y_n) = (x_1 + y_1, \ldots, x_n + y_n),$$
$$c(x_1, \ldots, x_n) = (cx_1, \ldots, cx_n).$$

Definition. For a positive integer k, and $v_1, \ldots, v_k \in \mathbf{R}^n$, we define the *span* of $v_1, \ldots, v_k$ to be

$$\operatorname{span}\{v_1, \ldots, v_k\} = \{c_1 v_1 + \cdots + c_k v_k \mid c_i \in \mathbf{R}\}.$$

1. Name one particular element v of $\mathbf{R}^6$, and explain why the definition of $\mathbf{R}^n$ tells you that v is an element of $\mathbf{R}^6$.

2. Let v_1, v_2, and v_3 be vectors in $\mathbf{R}^n$ for some positive integer n. Name one particular element w of $\operatorname{span}\{v_1, v_2, v_3\}$, expressed in terms of v_1, v_2, and v_3, such that $w \neq v_1$, $w \neq v_2$, and $w \neq v_3$. Explain why the definition of span tells you that w is an element of $\operatorname{span}\{v_1, v_2, v_3\}$.

3. The inclusion "$\operatorname{span}\{v_1, v_2, v_3\} \subseteq \operatorname{span}\{v_1, v_2, v_3, v_4, v_5\}$" is part of the statement of Theorem 1. Express this inclusion as an equivalent if-then statement.

4. Write out assumptions and conclusions for Theorem 1, putting as much as is logically possible into the assumptions. (See Section 8 of the proof notes on nested if-then statements.) Then use the definition of the span of a subset of $\mathbf{R}^n$ to rewrite those assumptions and conclusions. The result should essentially be one or two lines away from a complete proof.