

Math 108, problem set 01a
Due: Wed Feb 03
Last revision due: Mon Feb 22

Problems to be done but not turned in: (Smullyan) Ch. 3, problems 26–38; Ch. 8, problems 109–126.

Problems to be turned in:

Problems 1–4: For each of the theorems listed in these problems, separate the theorem into assumptions and conclusions. Do not try to prove any of the theorems. Except for the last theorem, you should have seen all of these theorems in calculus, but you should be able to do this **A/C** exercise even if you don't know or don't remember the definitions of some of the terms involved.

1. **Fundamental Theorem of Calculus.** If $F'(x) = f(x)$ on $[a, b]$, and f is continuous on $[a, b]$, then $\int_a^b f(x) dx = F(b) - F(a)$.
2. **Extreme Value Theorem.** If f is continuous on a closed interval $[a, b]$, then f attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ at some numbers c and d in $[a, b]$.
3. **Fermat's Theorem.** If f has a local minimum or local maximum at c , and $f'(c)$ exists, then $f'(c) = 0$.
4. **Theorem.** If v_1, v_2, v_3, v_4, v_5 are contained in \mathbf{R}^n , then

$$\text{span}\{v_1, v_2, v_3\} \subseteq \text{span}\{v_1, v_2, v_3, v_4, v_5\}.$$

Problems 5–8: Solve each puzzle, completely explaining/justifying your answer. Note that your answer should contain enough explanation to show that all other answers are incorrect (i.e., you can't just write down the final answer and expect full credit). See Smullyan Ch. 3 for an explanation of knights and knaves; see Smullyan Ch. 8 for a discussion of the truth value of an if-then statement.

In our problems, we will use slightly different terminology: People from **Tracy** always tell the **truth**, and people from **Livermore** always **lie**. In each problem, we assume that each of A and B (and C, if present) is either from Tracy or from Livermore. (Perhaps you meet them on the appropriate stretch of I-580.)

5. Suppose A and B say:

A: B is from Livermore.

B: Either A is from Tracy or I am from Tracy.

Can you tell where A is from? If so, where? Can you tell where B is from? If so, where?

(Continued on next page.)

6. Suppose A and B say:

A: If B is from Tracy, then I am from Livermore.

B: A is from [inaudible].

That is, you can't hear the last word that B says, but it's either Tracy or Livermore.
Can you tell where A and B are from? What did B say?

7. You come across A, B, and C, and A and B say:

A: Not all of us (A, B, and C) are from Livermore.

B: Not all of us are from Tracy.

Can you tell what A, B, and C are from? If so, where are they from?

8. You come across A, B, and C, and A and B say:

A: If B is from Livermore, then C is from Livermore.

B: If C is from Livermore, then A is from Livermore.

C: B and I are from different cities.

Can you tell what A, B, and C are from? If so, where are they from?