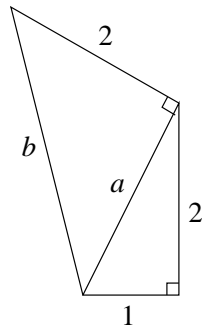


Some sample final exam questions
Math 10, Spring 2004

These are all of the questions from last semester's final exam that are related to material we have covered this semester. The actual exam was longer, as several questions have been omitted because they deal with material we did not cover. Conversely, since we have covered section 4.5 and 5.1–5.3, you should expect that material to appear on our exam. Also, this semester we covered Chapter 7 in more depth, so you should expect questions that are slightly harder than the Chapter 7 questions here.

Fundamentally, however, keep in mind that the last word as to what will appear on our exam is the homework and quizzes. Therefore, your study plan for this exam should be based on the homework and quizzes.

1. (10 points) Find the lengths a and b in the picture below (not to scale).



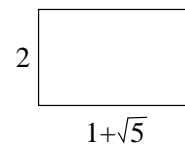
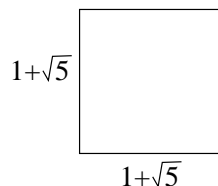
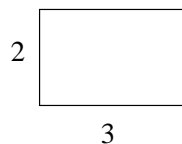
2. (10 points) The population of the United States is roughly 290 million, and the population of California is roughly 35 million. What is the probability that a random American lives in California? No explanation necessary.

3. (13 points) Which of the numbers $\sqrt{13}$, 3.5973 , $\frac{31\sqrt{3}}{7\sqrt{3}}$ are **rational**? Specifically, for each of the above numbers that is rational, explain how you know that the number is rational.

4. (12 points) Radio station KPOP has a strict playlist of 250 songs that it plays. One of their DJ's proposes that they have a "No-Repeat Friday," during which they will play at least 12 songs every hour, all day, without repeating any songs. Explain what's wrong with this idea.

5. (12 points) Which of the numbers 1000, 999, 23 are **not** prime? Specifically, for each of these numbers that is not prime, explain how you know that the number is not prime.

6. (12 points) For each of the rectangles below, determine if the rectangle is a Golden Rectangle. Justify your answer.



7. (14 points) Consider the following sets of numbers:

$$E = \{\text{even numbers}\} = \{2, 4, 6, 8, \dots\}$$

$$S = \{\text{nonnegative squares}\} = \{0, 1, 4, 9, \dots\}.$$

Do E and S have the same cardinality? If so, describe an explicit one-to-one correspondence between E and S ; if not, explain why not.

8. (14 points) You are given the chance to buy a \$2 lottery ticket that gives you a $1/5$ chance of winning \$3, a $1/20$ chance of winning \$10, and a $1/400$ chance of winning \$100. Would you buy this ticket? Explain your answer by computing the expected value of such a ticket.

9. (14 points) Suppose you roll a regular 6-sided die three times.

(a) What is the probability that you will roll a 6 three consecutive times?

(b) What is the probability that you will roll something besides a 6 at least once?

10. (14 points) By experimenting with examples in search of a pattern, find a formula for $F_n + F_{n-1} + F_{n-2}$ (i.e., the sequence $F_3 + F_2 + F_1$, $F_4 + F_3 + F_2$, $F_5 + F_4 + F_3$, etc., starting with $n = 3$). In other words, find a formula for the sum of a Fibonacci number and the two Fibonacci numbers that precede it in the Fibonacci sequence.

	$n = 1$	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 6$	$n = 7$
F_n							
$F_n + F_{n-1} + F_{n-2}$	---	---					

11. (14 points) In a few sentences, explain how to determine if a given real number is rational from its decimal expansion. (Assume that you have complete knowledge of the number's decimal expansion.)

12. (16 points) This question is about the proof of Cantor's theorem.

(a) Explain how to use a version of Cantor's construction to construct a "missing number" M whose digits after the decimal point are either 3 or 8. To illustrate this idea, apply your construction to the following list:

$$1 \leftrightarrow 8.22432982\dots$$

$$2 \leftrightarrow 2.23498729\dots$$

$$3 \leftrightarrow 5.29802929\dots$$

$$4 \leftrightarrow 3.23425000\dots$$

(b) What does this "missing number" M tell you about the possibility of finding a one-to-one correspondence from the natural numbers to the real numbers? Briefly **explain**.