

Sample exam 1
Math 10, Spring 2004

1. Suppose we have a sequence of numbers that starts 3, 1 and then continues by applying the same rule we used to make the Fibonacci sequence. Write down at least the first 7 terms of this new sequence, starting with 3, 1. No explanation necessary.
2. Pick a nonprime whole number between 10 and 20, and briefly explain why the number you chose is not prime.
3. There are 20,000 students at Cal State Rancho Carne, all of whom were born in the 20-year time span between Jan 1, 1958 and Dec 31, 1977. Can you be certain that at least two of them were born on the same day? Briefly explain your answer.
4. A boy and a girl are sitting on a bench. "I'm a boy," says the one with black hair. "I'm a girl," says the one with red hair. You know that at least one of them is lying. What is the hair color of the girl?
5. You are trying to cross a river with your favorite wolf, favorite goat, and favorite cabbage. You have a boat that is just big enough to hold you and any one of those three items, but no more. If you leave the wolf alone with the goat, it will eat the goat, and if you leave the goat alone with the cabbage, it will eat the cabbage. How do you get all three items across the river? Explain your answer or draw a diagram that shows how you can do this.
6. By experimenting with examples in search of a pattern, find a formula for $F_{n+2} - F_n$ (i.e., the sequence $F_3 - F_1, F_4 - F_2, F_5 - F_3, \dots$). In other words, find a formula for the difference between a Fibonacci number and the second Fibonacci number before it.

If it helps, you can fill in the following table:

F_{n+2}								
F_n								

Note: If you don't see how to express the pattern you see in a formula, at least explain it in words.

7. If p is a prime larger than 5, is it possible that when you divide p by 6, the remainder is 2? First try examples in search of a pattern, and then for full credit, briefly **explain** the pattern you see.

Prime $p > 5$							
Remainder when you divide p by 6							