Recent approaches for the analysis of nonstationary time series with applications to biomedical signal processes

Raquel Prado
Associate Professor of Statistics
Department of Applied Mathematics and Statistics
University of California Santa Cruz

December 8, 2010

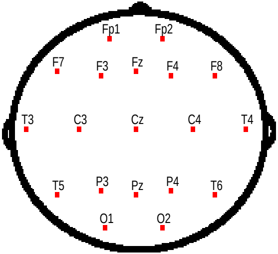
Outline

EEG monitoring of ECT

- Electroconvulsive therapy is a treatment for major depression.
- EEG has been used as one of the primary methods of ECT monitoring.
- Study was conducted at Duke University by A. Krystal and collaborators.
- Maximize the therapeutical efficacy of the treatment while minimizing the side effects. Clinically relevant question: Are there any EEG features that can be associated with treatment efficacy?

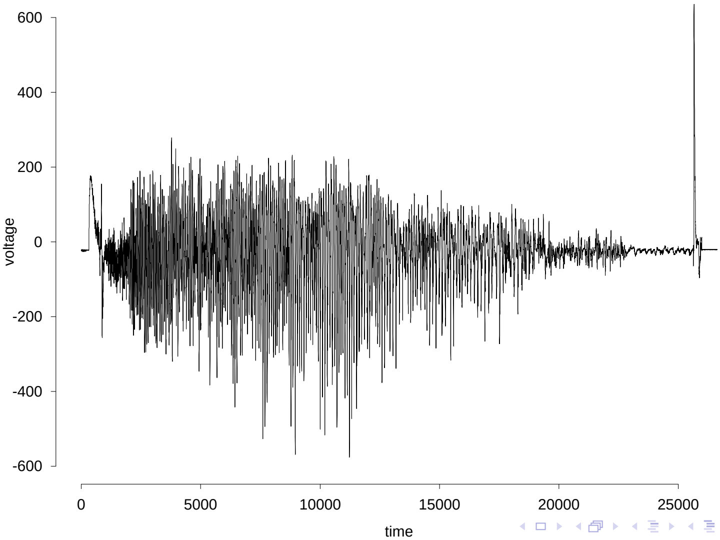
The Ictal-19 data set

- 19 EEG channels recorded during ECT seizure



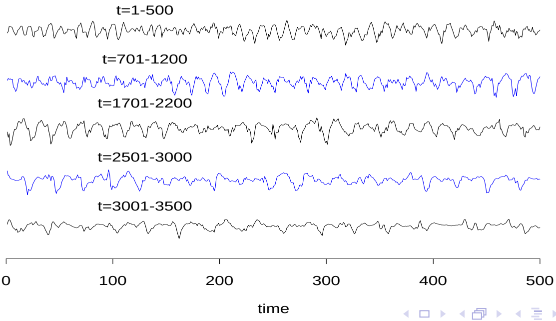
- **Aim:** characterize the redundancy among the 19 channels as a preliminary step towards a better understanding of the physiology underlying the effectiveness of ECT.

ECT Data: Single Channel



EEG monitoring of ECT

EEG phases during ECT monitoring: *baseline pre-ictal* $\rightarrow$ *electrical stimulus* (blocked) $\Rightarrow$ *pre-ictal* $\rightarrow$ *epileptic recruiting* $\rightarrow$ *polyspike* $\rightarrow$ *polyspike and slow wave* $\rightarrow$ *termination* $\rightarrow$ *post-ictal*



Multi-patient data

Further data analyses: latent components in the 2-channel EEG data set

– Many multiple series

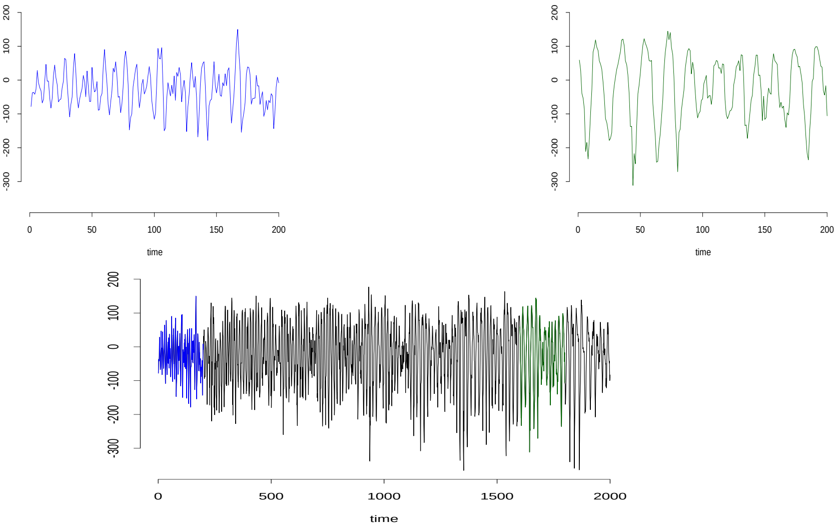
– One seizure: 2 channels, 256 Hz (20-26,000 observations each)

– *Repeat:*

different ECT treatment (level, duration of stimulus, drugs, ...)

Clinical issues: *treatment effects on seizure patterns?*

Stationarity vs Nonstationarity



Autoregression (AR)

$$y_t \;=\; \phi_1 y_{t-1} + \phi_2 y_{t-2} + \ldots + \phi_p y_{t-p} + \epsilon_t,$$

with $\epsilon_t \sim N(0, v)$.

AR characteristic polynomial

$$\Phi(u) = 1 - \phi_1 u - \phi_2 u^2 - \ldots - \phi_p u^p.$$

If all the roots of $\Phi(u)$ lie outside the unit circle, i.e., if $\Phi(u) = 0$ only when $|u| > 1$, the AR(p) process above is *stationary*.

Conditional Bayesian inference in AR models

Bayesian inference requires prior distributions on the model parameters $\phi = (\phi_1, \ldots, \phi_p)'$ and v. The prior and the likelihood are then combined to obtain the posterior distribution via:

$$p(\phi, v|\mathbf{y}) \propto p(\mathbf{y}|\phi, v) \times p(\phi, v),$$

or via

$$p(\phi, v|\mathbf{y}) \propto p(\mathbf{y}_{(p+1):n}|\mathbf{y}_{1:p}, \phi, v) \times p(\phi, v)$$

in the conditional case.

Priors for AR models

- Conjugate priors
  - Multivariate normal prior on $(\phi|v)$: $\phi \sim N(\mathbf{0}, v\Sigma)$.
  - Inverse-Gamma prior on v or equivalently, Gamma prior on the precision $1/v$.

  This is computationally easy but does not restrict the AR coefficients to the stationary region.
- Non-conjugate priors. Many alternatives...

Decompositions: AR case

If $y_t \sim AR(p)$, it is possible to show that:

$$y_t = \sum_{j=1}^{c} z_{t,j} + \sum_{j=2c+1}^{p} x_{t,j}$$

- $z_{t,j} \sim$ quasi-periodic ARMA(2,1)
  - "sinusoid" with randomly time-varying amplitude and phase and
  - constant *characteristic frequency* ω_j *and modulus* r_j
- $x_{t,j} \sim$ AR(1)

Pros & Cons of AR models

Advantages of AR models:

- *Linear* $\Rightarrow$ (depending on the priors) these models are easy to fit. In particular, they can be fitted in real time.
- Some functions of the AR parameters are interpretable. E.g., for quasiperiodic processes it is possible to describe activity in various frequency bands.

Disadvantages:

- Not appropriate for nonstationary time series.

Better Models...

Q. What can be better than an autoregressive model? (and still relatively simple...)

- An autoregressive model with time-varying parameters
- Two (or more) autoregressions....

Better Models...

Q. What can be better than an autoregressive model? (and still relatively simple...)

- An autoregressive model with time-varying parameters
- Two (or more) autoregressions....

TVAR Models

Time-varying autoregressions

$$
\begin{aligned}
y_t &= \sum_{i=1}^{p} \phi_{t,i} y_{t-i} + \epsilon_t, \quad \epsilon_t \sim N(0, v_t), \\
\phi_{t,i} &= \phi_{t-1,i} + \omega_{t,i} \quad \omega_{t,i} \sim N(0, w_{t,i}) \\
v_t &= \delta_v v_{t-1}/\eta_t, \quad \eta_t \sim Beta(a_t, b_t).
\end{aligned}
$$

Advantages:

- Locally linear $\Rightarrow$ (again, depending on the priors) models are easy to fit. Kalman filters can be used to fit the models in real time.
- Interpretable (locally).

Disadvantages:

- Do capture some types of nonstationarities but not all.

Decompositions: TVAR case

If $y_t \sim TVAR(p)$, it is possible to show that:

$$y_t = \sum_{j=1}^{c} z_{t,j} + \sum_{j=2c+1}^{p} x_{t,j}$$

- $z_{t,j} \sim$ quasi-periodic TVARMA(2,1)
  - "sinusoid" with randomly time-varying amplitude and phase and
  - time-varying *characteristic frequency* $\omega_{t,j}$ *and modulus* $r_{t,j}$
- $x_{t,j} \sim$ TVAR(1)

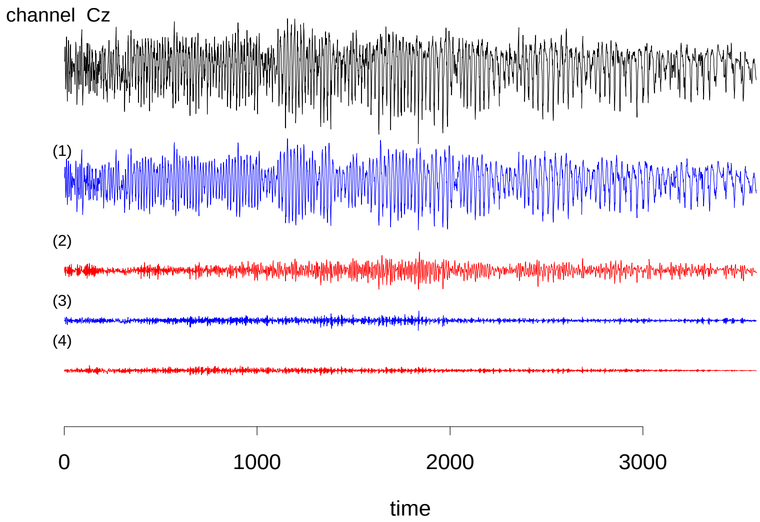
Note: Interpretation is approximate.

Single channel decompositions

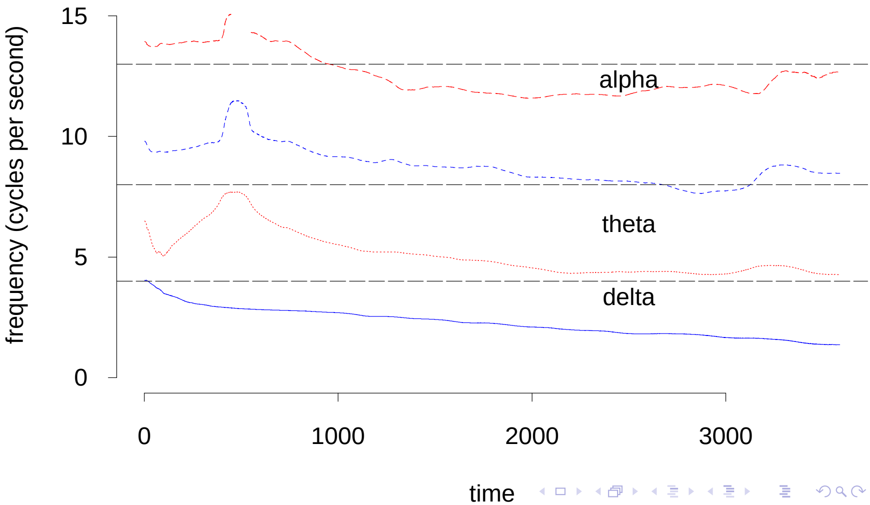
Decomposition results: based on posterior mean of ϕ_t at each time t.

- Trajectories of estimated latent components
- Trajectories of characteristic frequencies and moduli

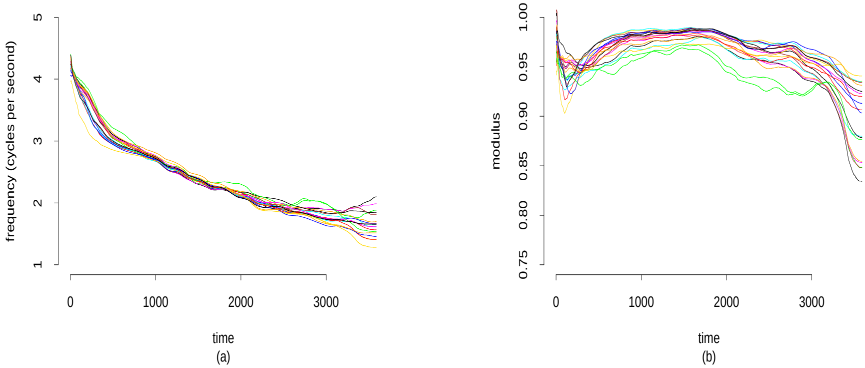
Decomposition of channel Cz



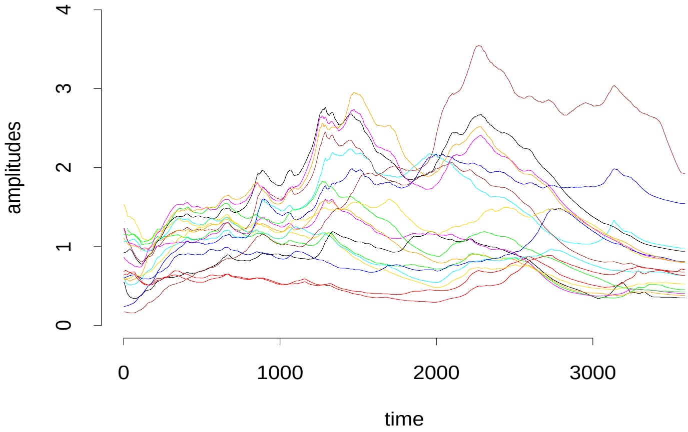
Frequency trajectories in channel Cz



Latent Components: 19 channels

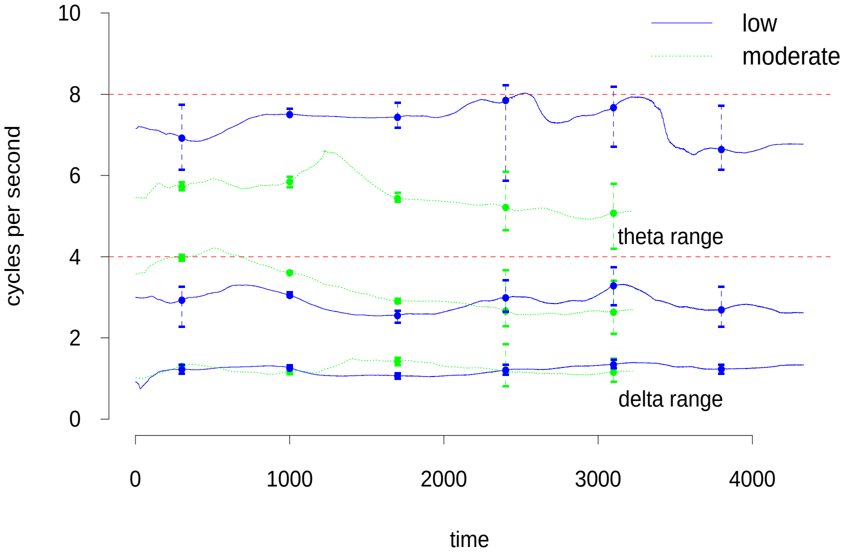


Latent Components: 19 channels

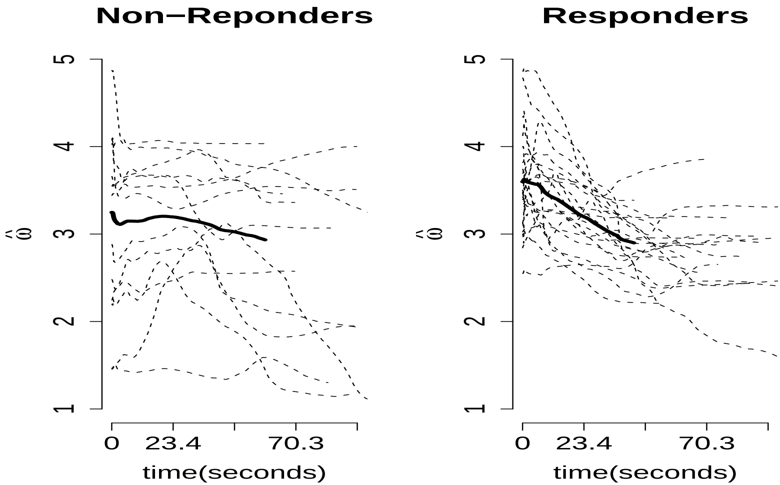


We can also use the results obtained from the TVAR approach to comparing treatments...

Results


10
8
6
4
2
0
cycles per second
low
moderate
theta range
delta range
0
1000
2000
3000
4000
time

Results


Non−Reponders
Responders
5
4
3
2
1
$\hat{\omega}$
0
23.4
70.3
time(seconds)

Outline

Cognitive fatigue data

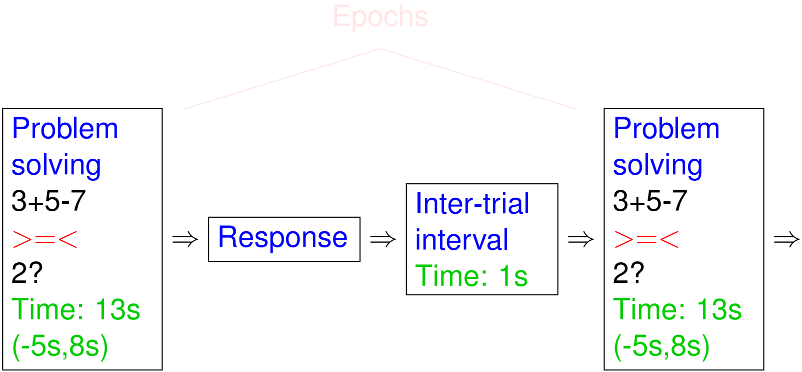
- Data collected in EEG lab at NASA Ames by L. Trejo and collaborators
- 30-channel EEGs were recorded from 16 subjects who performed up to 180 min of non-stop computer-based mental arithmetic
- Observed behavior included ratings of activity and alertness from videotape recordings of each participant's performance
- Performance measures: response time and response accuracy
- Physiological measures: derived from EEGs and EOGs

Cognitive fatigue data

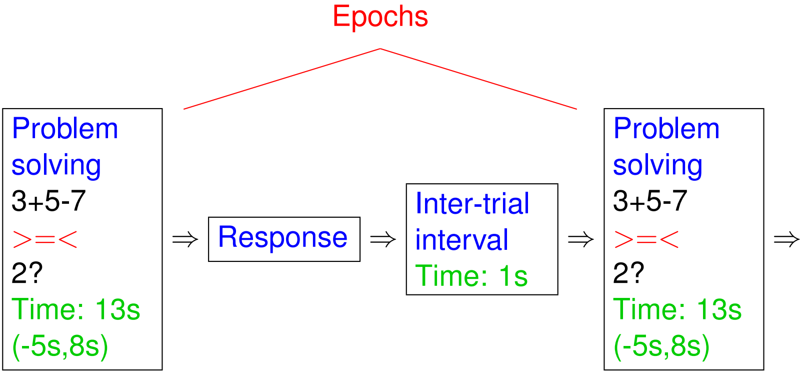
EEGs were...

- submitted to algorithms for detection & elimination of artifacts
- epoched around the stimulus (i.e., from -5s pre-stimulus to +8 post-stimulus)
- low passed filtered (50 Hz; zero phase shift; 12 dB/octave roll off)

Cognitive fatigue data



Cognitive fatigue data



Cognitive fatigue data

For each individual and each channel we have a collection of "consecutive" epochs...

Epochs

An epoch is a time series of 1664 observations. It corresponds to 13 seconds of recording, with 5s prior to the stimulus and 8s after the stimulus. The sampling rate is then 128 Hz.

Note: total number of epochs varies with the subject.

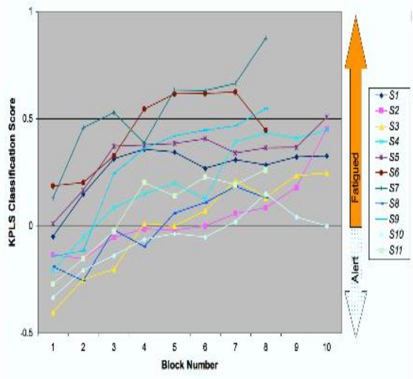
Cognitive fatigue data

Goals

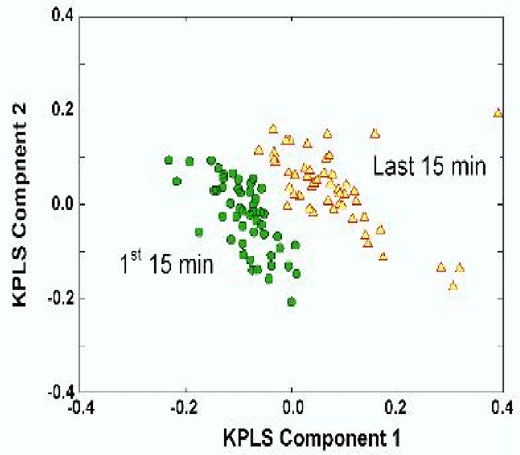
- Can we detect fatigue from EEG data?
- If so, what characterizes fatigue?
- Are there two or more mental states of alertness?
- Long term: automatic system for fatigue detection from physiological signals

Previous Analyses: Classification via KPLS-DLR

Previous analyses include kernel partial least squares decomposition of multi-channel EEG spectra coupled with a discrete-output linear classifier (KPLS-DLR, Rosipal, Trejo & Matthews, 2003)



Classification via KPLS-DLR



Motivation for other model-based analyses

- We would like a framework that allows us to interpret the results in terms of parameters that are meaningful (e.g., brain waves and brain activity)
- Include prior information collected from previous experiments
- We want to answer questions such as, what is the probability that a given subject is fatigued at time t?

Descriptive analysis: subject skh

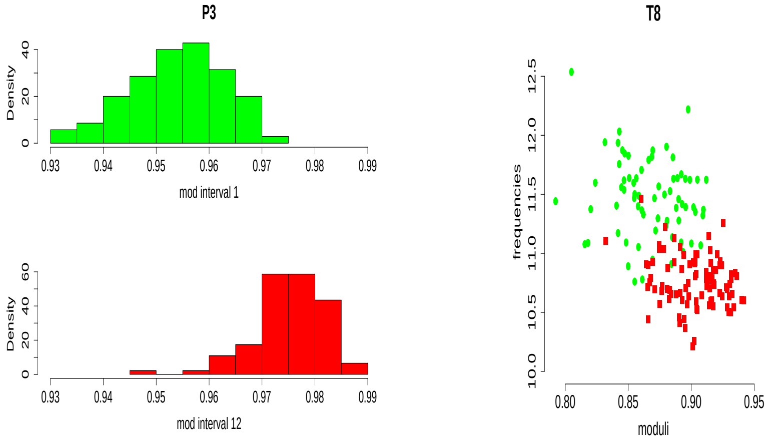
Let $y_{t,q,j}$ be the t-th observation of epoch q for channel j, with $t = 1 : T$, $q = 1 : Q$, and $j = 1 : J$. For subject **skh** we have $T = 1664$, $Q = 864$ and $J = 30$.

$$y_{t,q,j} = \sum_{i=1}^{p} \phi_{i,q,j} y_{t-i,q,j} + \epsilon_{t,q,j}, \quad \epsilon_{t,q,j} \sim N(0, v_{q,j}).$$

We look at the posterior distribution of the reciprocal roots of the polynomial

$$\Phi_{i,q,j}(u) = 1 - \phi_{1,q,j} u - \ldots - \phi_{p,q,j} u^p.$$

Preliminary results: skh



MAR

We assume that with probability $p_q(k)$ each epoch q is described by the autoregressive model $\mathcal{M}_q(k)$, with

$$\mathcal{M}_q(k): \qquad y_{t,q} = \phi_1^{(k)} y_{t-1,q} - \ldots - \phi_p^{(k)} y_{t-p,q} + \epsilon_{t,q}^{(k)},$$

with $\epsilon_{t,q}^{(k)} \sim N(0, v)$.

The models $\{\mathcal{M}_q(1), \ldots, \mathcal{M}_q(K)\}$ represent K brain states.

MAR: Further Model Structure

Let $\mathcal{D}_{q-1} = \{\mathcal{D}_0, \mathbf{y}_{1:(q-1)}\}$.

Prior and posterior at epoch q

$$\begin{array}{ccc} & \mathbf{y}_q & \\ & \downarrow & \\ \pi_q(k) \equiv Pr[\mathcal{M}_q(k)|\mathcal{D}_{q-1}] & \Longrightarrow & p_q(k) \equiv Pr[\mathcal{M}_q(k)|\mathcal{D}_q] \end{array}$$

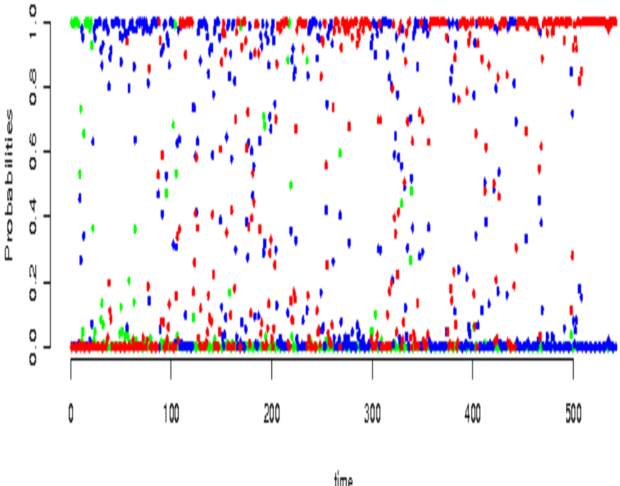
Transition probabilities

$$Pr[\mathcal{M}_q(k)|\mathcal{M}_{q-1}(i), \mathcal{D}_{q-1}] = Pr[\mathcal{M}_q(k)|\mathcal{M}_{q-1}(i), \mathcal{D}_0] \equiv \pi(k|i)$$

MAR: On-line Posterior Inference

- Non-conjugate prior distributions are placed on the AR coefficients $\Rightarrow$ Posterior inference is not available in closed form.
- Regardless of the priors, approximations are used so that the number of components in the mixture model does not increase over time.
- On-line inference: (a) Approximations (b) Sequential Monte Carlo Algorithms.

Fatigue data: subject rwc, channel P8



Outline

Remarks

- TVAR and MAR models are useful to describe complex signals such as EEGs
- Computational challenges:
  - Large-dimensional data sets
  - Models with a large number of parameters
  - Non-conjugate priors
  - On-line inference
- Future: Multivariate approaches

AMS Department

The Applied Mathematics and Statistics Department is part of the Baskin School of Engineering.
Other Departments at Baskin Engineering:

- Biomolecular Engineering
- Computer Engineering
- Computer Science
- Electrical Engineering
- Technology and Information Management

Meet our faculty

David Draper (Statistics)

- Bayesian methods
- Bayesian nonparametrics
- Stochastic models in medical and social sciences

Meet our faculty

Thanasis Kottas (Statistics)

- Bayesian nonparametrics
- Point-process modeling
- Survival analysis

Meet our faculty

Herbie Lee (Statistics)

- Spatial inverse problems
- Statistical computing

Meet our faculty

Abel Rodriguez (Statistics)

- Bayesian nonparametrics
- Econometrics and finance
- Time series and spatial models

Meet our faculty

Bruno Sanso (Chair, Statistics)

- Bayesian spatio-temporal modeling
- Environmental applications (climate models)

Meet our faculty

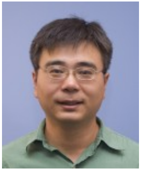
Nic Brummell (AM) and Pascale Garaud (AM).
Research areas: supercomputing, non-linear PDEs, magnetohydrodynamics, astrophysical and geophysical fluid dynamics

Meet our faculty

Marc Mangel (AM), Hongyun Wang (AM).
Research areas: mathematical biology, ecology and evolutionary biology and biophysics (protein motors).


Meet our faculty

Dejan Milutinovic (AM), Qi Gong (AM).
Research areas: control theory and applications, stochastic dynamical systems and multi-agent systems.

Meet our faculty

Eric Anderson (AM, S) and **Robin Morris (S)**
Research areas: statistical methods in fisheries management and ecology, Bayesian analysis of astrophysical, engineering and remote sensing data.

Meet our alumni

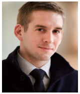
Robert Gramacy. Assistant Professor of Econometrics and Statistics, The University of Chicago Booth School of Business.

*Bayesian Treed Gaussian Process Models*

Dan Merl. Researcher, Statistics Group at Lawrence Livermore National Lab.

*Detecting natural selection via Bayesian GLMs*

Matt Taddy. Assistant Professor of Econometrics and Statistics, The University of Chicago Booth School of Business.

*Bayesian nonparametric analysis of Poisson point processes*

Meet our alumni

- **Michael Derouen.** MSc Statistics. *Bayesian Flaw Detection in 3D Point Cloud Data.* Applied Signal Technology (Bay Area).
- **Xing Ji.** MSc in Statistics. *A Bayesian Modeling Application to Estimating the Climate Change Impact on the Presence of Oaks in California.* Risk Management Department, e-Bay Paypal.
- **Milovan Krnjajic.** PhD Statistics. *Contributions to Bayesian Statistical Analysis: Model Specification and Nonparametric Inference.* National University of Ireland.
- **Juancarlos Laguardia.** MSc Statistics. *Contributions to Biostatistics: Likelihood Ratios and Vital Signs.* Consulting Analyst, Division of Research, Kaiser Permanente.

Meet our alumni

- **Chris Wong.** MSc Statistics. *Forecasting support burden for the CISCO 2006 routers.* CISCO.
- **Yuzheng Zhang.** MSc Statistics. *EEG-based identification of mental fatigue.* Statistics Research Associate, Fred Hutchinson Cancer Center.
- **Weining Zhou.** PhD Statistics. Dissertation: *Analyzing computer simulation experiments using process convolutions.* Metrics Interpretation and Analysis Group at Yahoo!

Graduate Programs

- PhD program in Statistics
- PhD program in Applied Mathematics
- MSc in Statistics
- MSc in Applied Mathematics

Deadline for Fall 2011: January 4, 2011. Go to
`www.soe.ucsc.edu/advising/graduate/admissions`

Statistics PhD

Financial Support. AMS attempts to provide financial support, in the form of Fellowships, Teaching and Research Assistantships, to all students admitted with priority typically given to Ph.D. students. Thanks to the support of the National Science Foundation the department is also able to offer scholarships through the **S-STATSMODEL (Scholarships in Statistics and Stochastic Modeling)** program.

This program aims to increase the number of academically talented, financially disadvantaged students in AMS. It provides need-based scholarship awards of up to $10,000 each year for up to two years to a cohort of three to four students.

S-STATSMODEL fellowships

Application Procedure (go to `http://www.ams.ucsc.edu/academics/graduate`**)**

1. Complete an application to the AMS department through the School of Engineering admissions website.
2. Complete a free application for Federal Student Aid (FAFSA).
3. Complete a S-STATSMODEL application form (including a personal statement, which should be different from the one used for AMS departmental application and should specifically address the points mention in the form).

Deadline: March 1st, 2011.

Contact Information

- Bruno Sanso (Chair): bruno@ams.ucsc.edu
- Nic Brummell (Graduate Director): brummell@soe.ucsc.edu
- Raquel: raquel@ams.ucsc.edu
- SOE website: www.soe.ucsc.edu
- AMS website: www.ams.ucsc.edu

Feel free to e-mail any faculty member if you are interested in his/her research!