Are the Oceans Warming?
Statistical Models for Ocean Temperatures

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In collaboration with Ricardo Lemos, Lisboa, Portugal.

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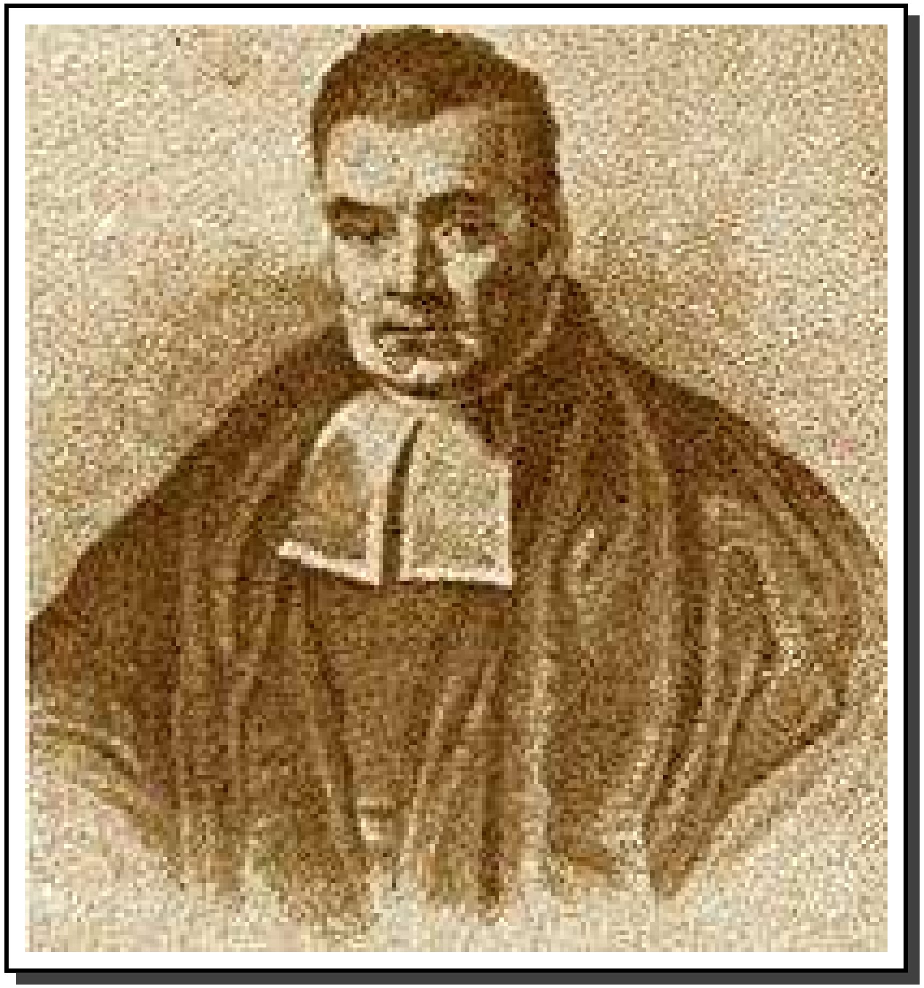
Thomas Bayes: 1702 – 1761



$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Hierarchical Models

A useful framework for statistical models applied to problems in the physical sciences is given by hierarchical models with the levels:

Observational Model:

$$p(Y|\theta)$$

measurement error

Process Model:

$$p(\theta|\lambda)$$

process variability

Parameters Model:

$$p(\lambda)$$

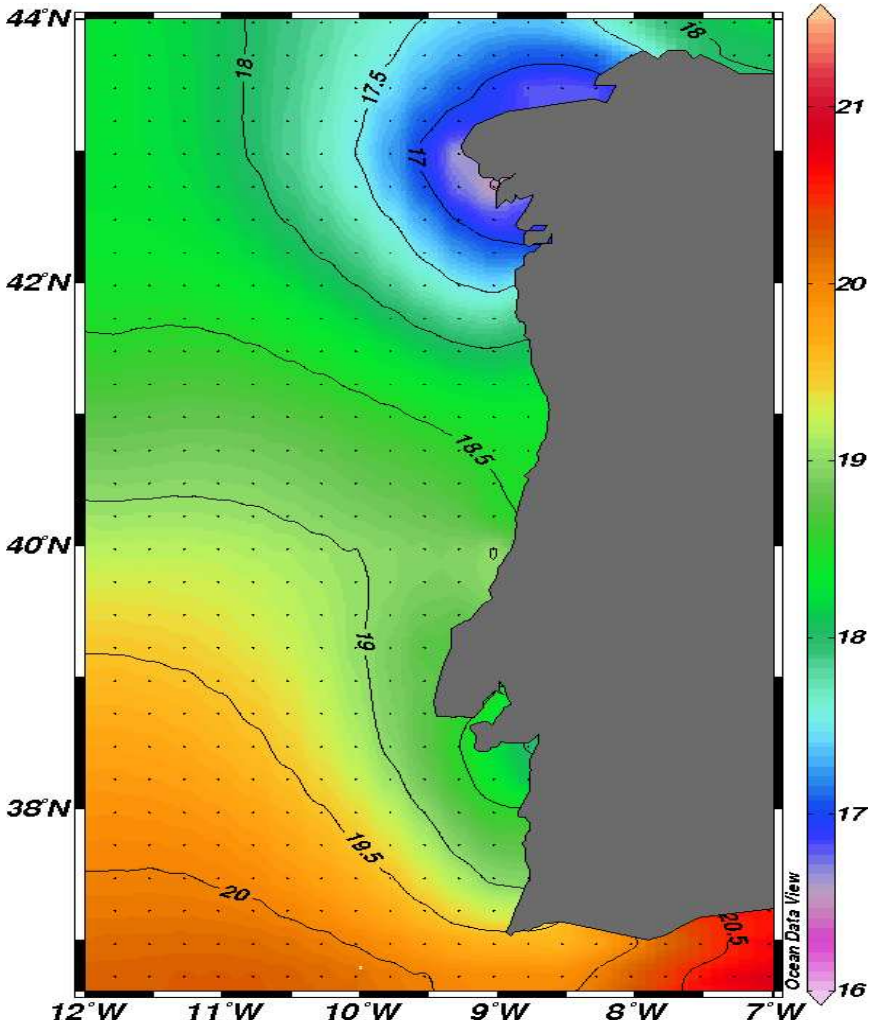
prior information

To learn about θ we obtain the posterior distribution $p(\theta|Y)$ using Bayes theorem.

Unfortunately, oftentimes the posterior distribution is intractable. A popular approach is to obtain samples from $p(\theta|Y)$ and use them for inference on θ.

A very common method for sample-based inference is to simulate a Markov chain whose equilibrium distribution is $p(\theta|Y)$. After running the chain for a "burn in" period necessary to reach steady state the samples are collected and used for inference.

July average SST from WOA.



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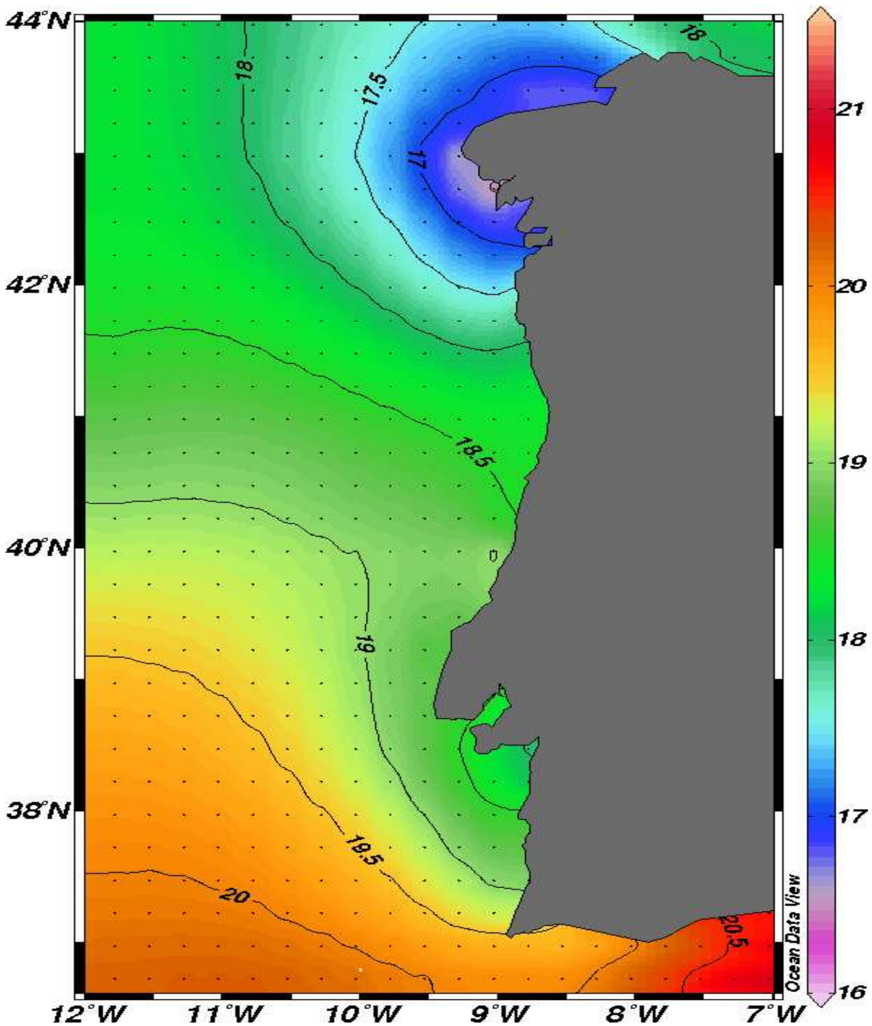

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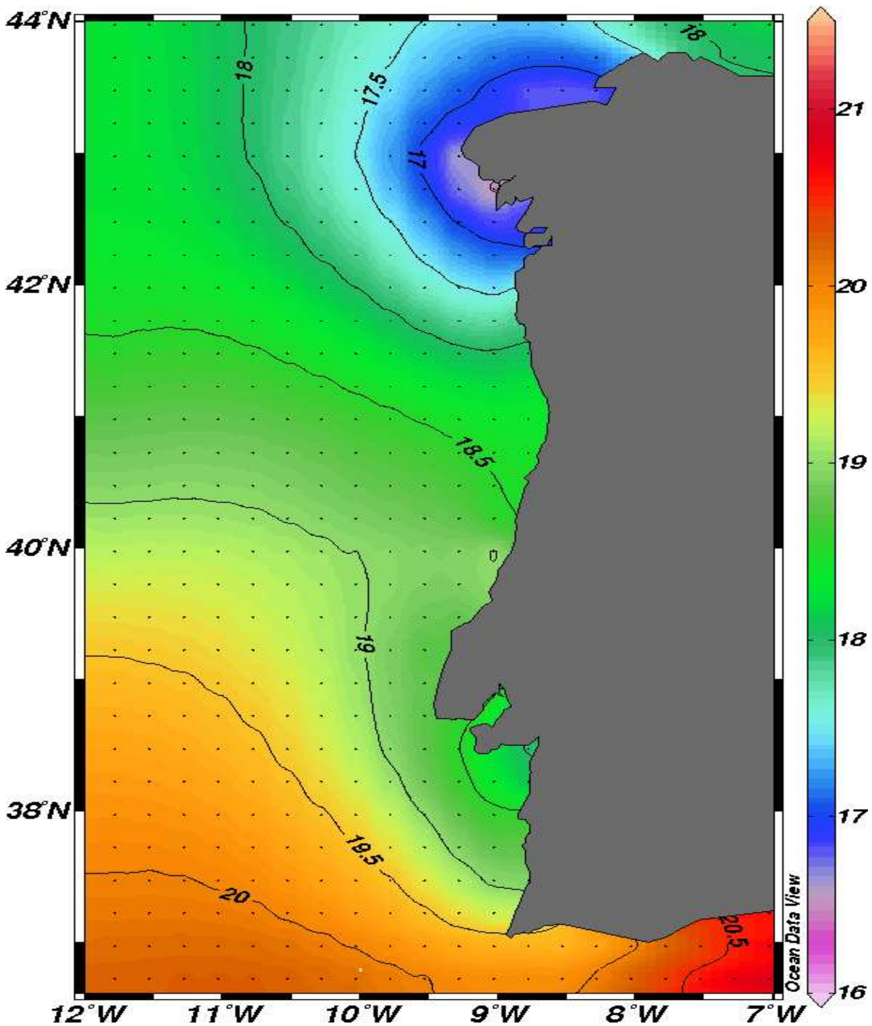

- Climatologies of ocean temperatures describe the mean state of the ocean using historical records.
- They are used to understand the properties, distribution and circulation of water masses.
- They are also used to calibrate remote sensors and for the spin up, forcing, relaxation and validation of numerical models.

JULY AVERAGE SSS FROM WOA.

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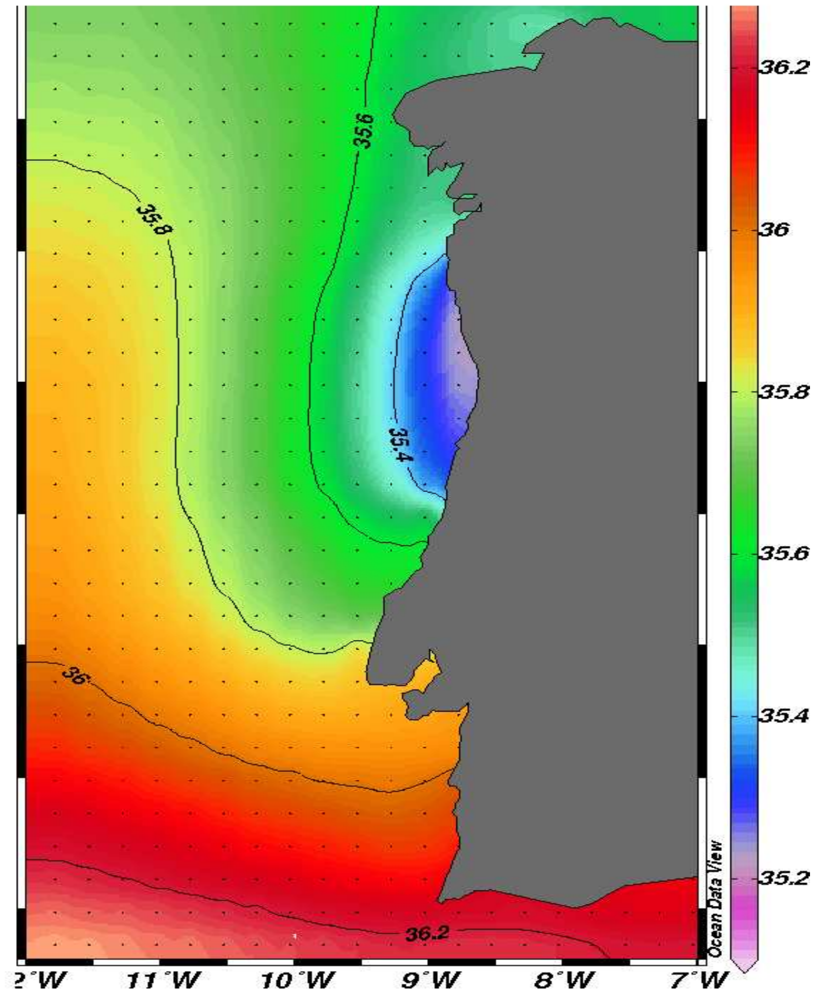

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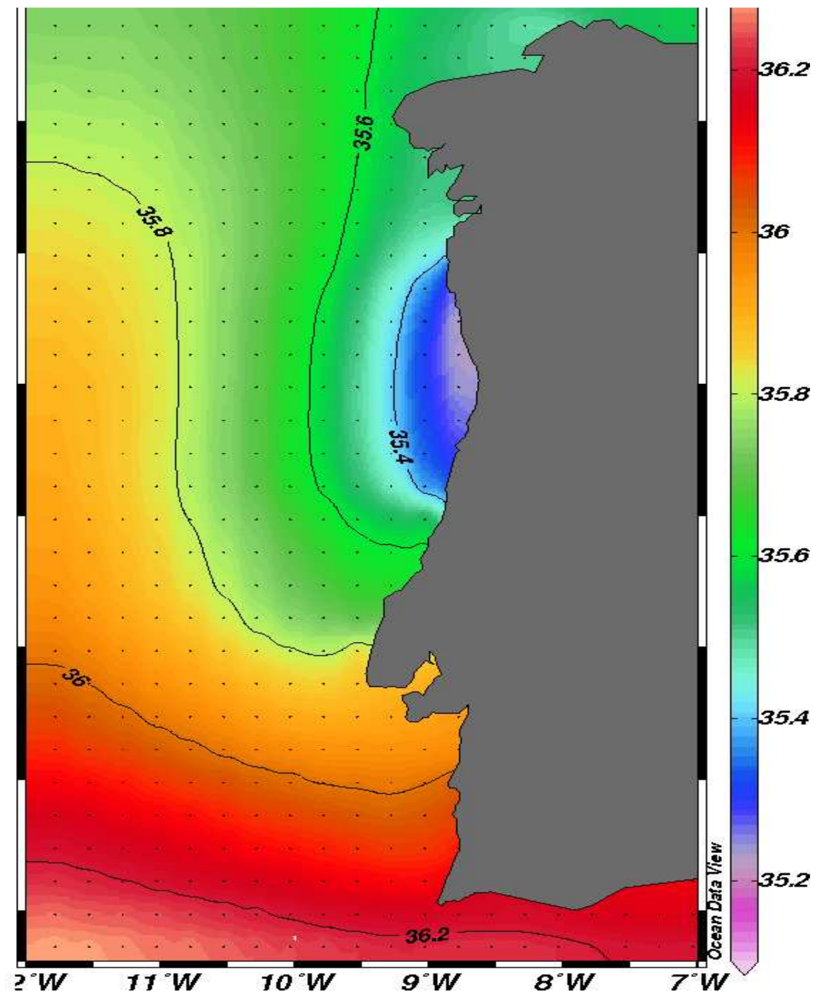
- Finally, there is keen interest in the detection of long-term changes in ocean properties.



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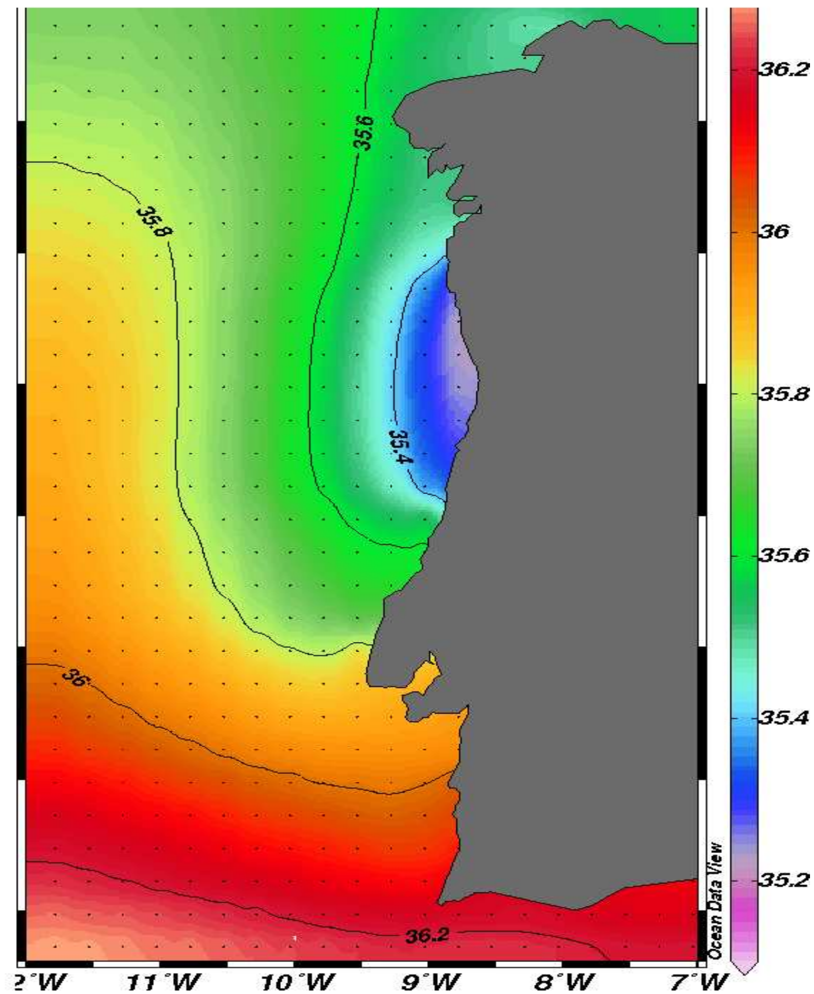
- In addition to the climatic mean, scientists are interested in the variability around the mean, or anomalies.
- Finally, there is keen interest in the detection of long-term changes in ocean properties.
- At present, one of the standard climatological products is provided by the National Oceanographic Data Center (NODC) and is the World Ocean Atlas 2001, version 2.



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- The method must be useful in a large geographical domains and long time frames.
- Observational errors should be accounted for.

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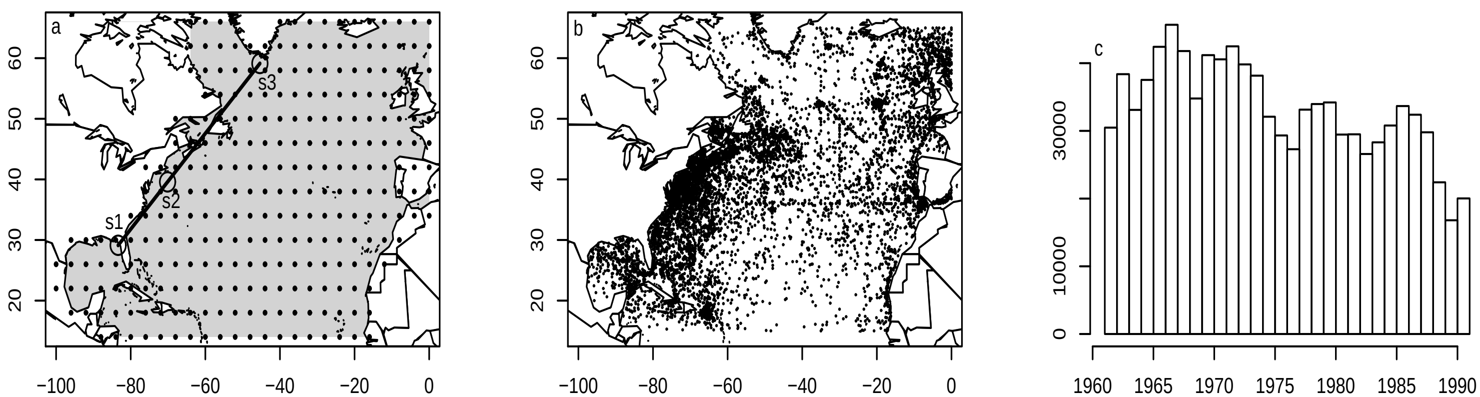
• To incorporate different sources information, including structural knowledge.

• To produce probabilistic measures of uncertainty.

The domain S (gray area) and the grid J (bullets; $r_J = 4°$) used for the convolving process. A transect with three "case study" points. A random sample of 1% of the data. Temporal distribution of the data.



We use data from the NODC World Ocean Database 2005, collected with four types of instruments between 1961 and 1990.

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Data set name	no. obs.	variance
OSD	261,172	6.25×10^{-2}
CTD	29,879	6.25×10^{-6}
XBT	419,263	2.5×10^{-3}
MBT	439,783	2.025×10^{-1}

Discrete Process Convolutions

To reach the above goals, we build flexible spatio-temporal processes based on representing a Gaussian process as the convolution of simple processes over a grid.

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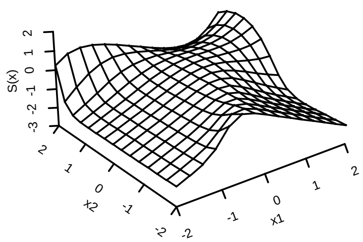
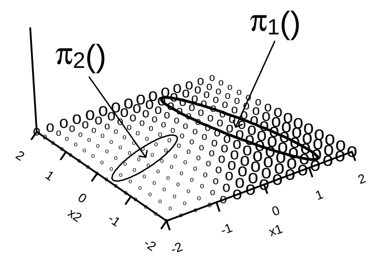
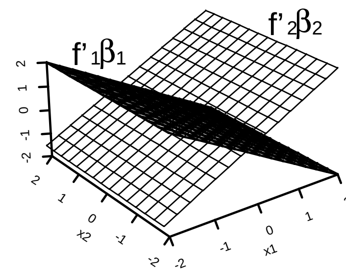
For any point $\boldsymbol{s}$ in space S, say $\theta(\boldsymbol{s})$, we write

$$\theta(\boldsymbol{s}) = \sum_{\boldsymbol{j} \in J} K[\boldsymbol{s} - \boldsymbol{j}, \boldsymbol{\omega}(\boldsymbol{s})]\psi(\boldsymbol{j})$$

J is a grid in S. $K[\cdot, \boldsymbol{\omega}]$ is a kernel depending on a parameters $\boldsymbol{\omega}$, and $\psi(\boldsymbol{j})$ is a random field with a simple correlation structure.We term this a Discrete Process Convolution.

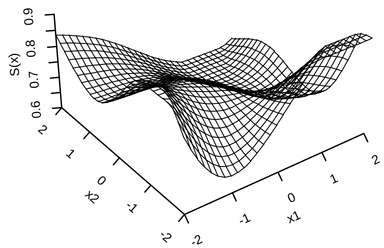
Spatial Process	Weighting Kernels	Component Surfaces
		

$$S(s;\boldsymbol{\beta}) \quad = \quad \sum_{j=1}^{2} \pi_j(s) \quad \times \quad \boldsymbol{f}_j'(s)\beta_j$$

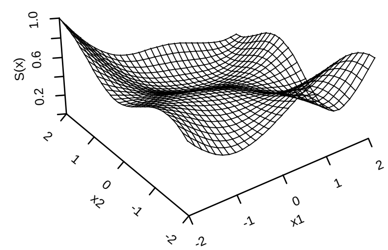
Spatial process generated from a mixture with $J = 2$. $\pi_j(\cdot)$ are weighting kernels. $\boldsymbol{f}_j'(s)\boldsymbol{\beta}_j$ are linear regression surfaces.

Dynamic Conditionally Linear Models

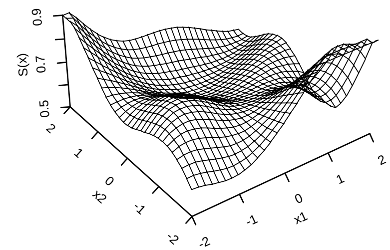
Time 1



Time 2



Time 3



$$S(s, \boldsymbol{\beta}_1) \quad \longrightarrow \quad S(s, \boldsymbol{\beta}_2) \quad \longrightarrow \quad S(s, \boldsymbol{\beta}_3)$$

A time series of spatial mean fields obtained through a locally-weighted mixture of $J = 6$ components with fixed weighting kernels and dynamic regression coefficients.

Model for North Atlantic SSTs

The SST observation $x_{i,m,y}(\boldsymbol{s})$ collected with instrument $i = 1, \ldots, 4$, in month m, year y and location $\boldsymbol{s}$ follows

$$x_{i,m,y}(\boldsymbol{s}) \sim N\left(\theta_{m,y}(\boldsymbol{s}), \tau_i^2\right).$$

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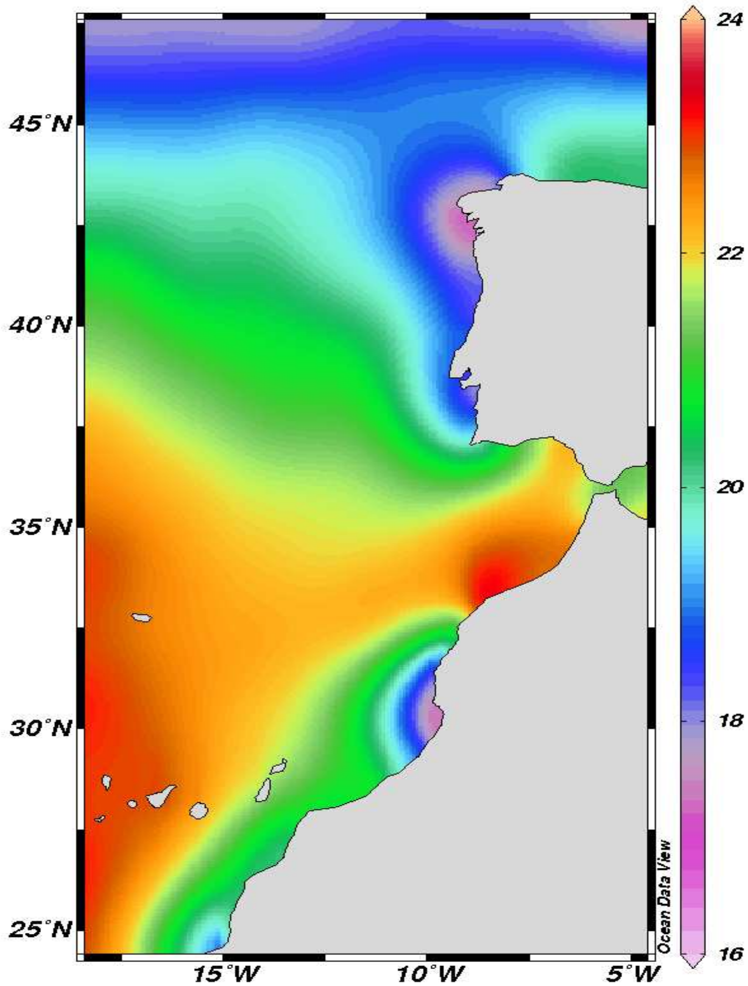
$$\theta_{m,y}(\boldsymbol{s}) \sim N\Big(\sum\nolimits_{\boldsymbol{j}} K[\boldsymbol{s}-\boldsymbol{j}, \boldsymbol{\Lambda}(\boldsymbol{s})]\left(\alpha(\boldsymbol{j}) + \boldsymbol{\beta}_t(\boldsymbol{j})\boldsymbol{w}_t^T + \eta(\boldsymbol{j})(t-180)\right), \Phi(\boldsymbol{s})^2\Big).$$

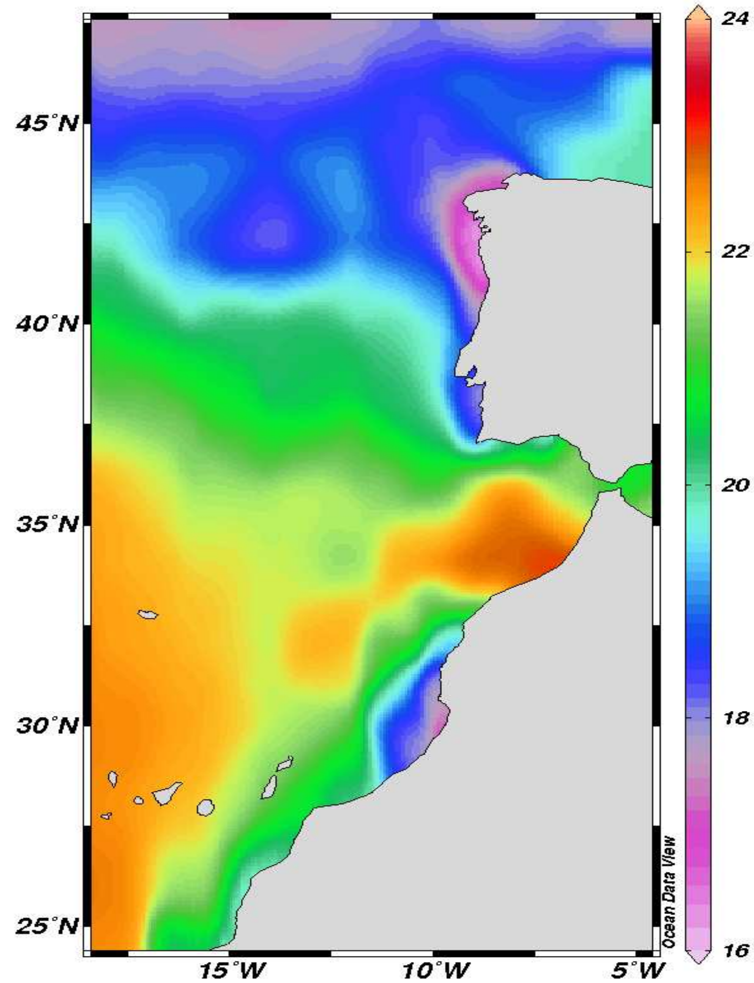
$$t = m + 12(y-1961) \quad \text{and} \quad \Phi(\boldsymbol{s})^2 = \sum\nolimits_{\boldsymbol{j}} K[\boldsymbol{s}-\boldsymbol{j}, \boldsymbol{\Omega}(\boldsymbol{s})] \exp\left(\sigma(\boldsymbol{j})\right).$$

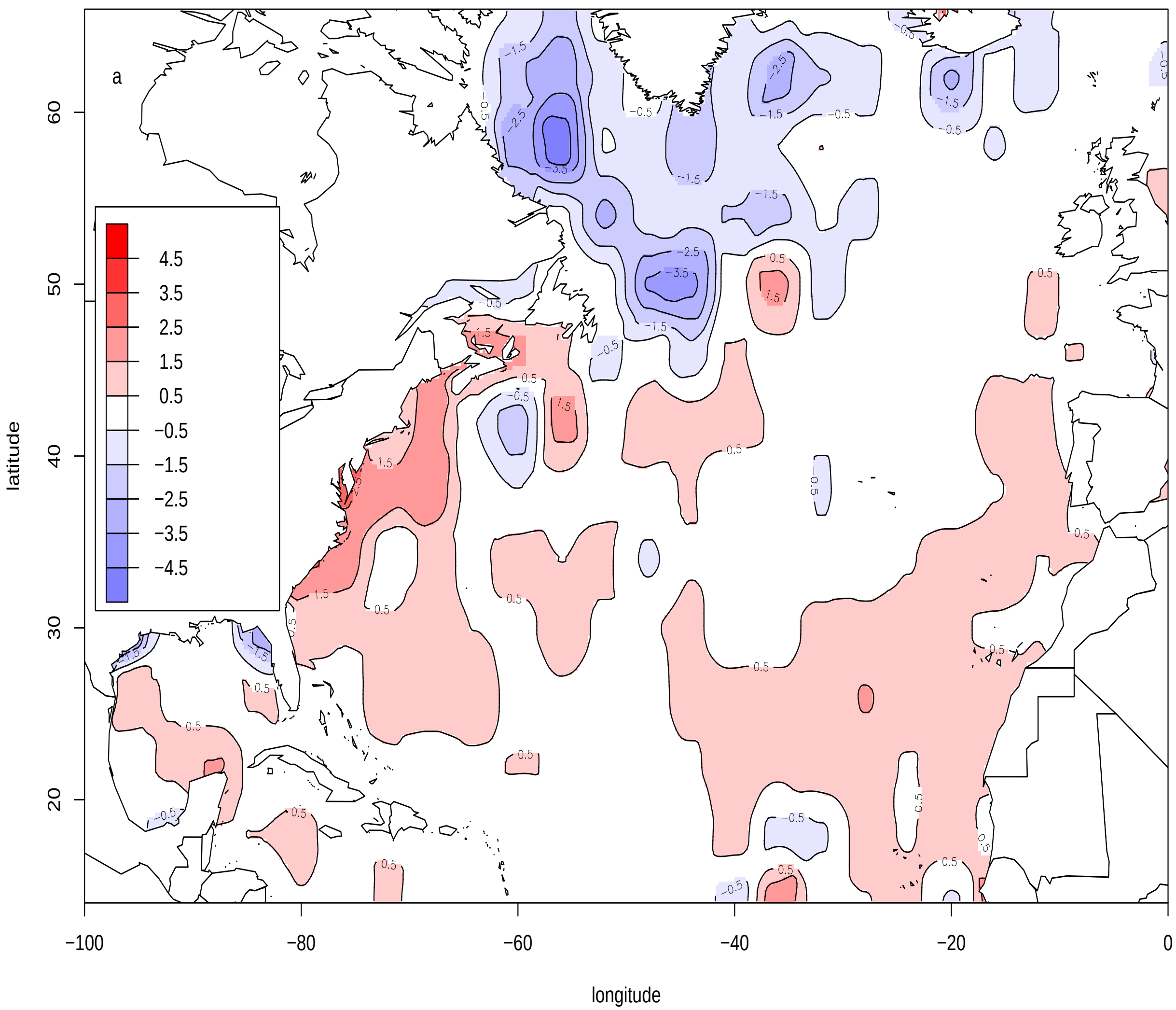
Results: August Climatology

August SST from WOA01



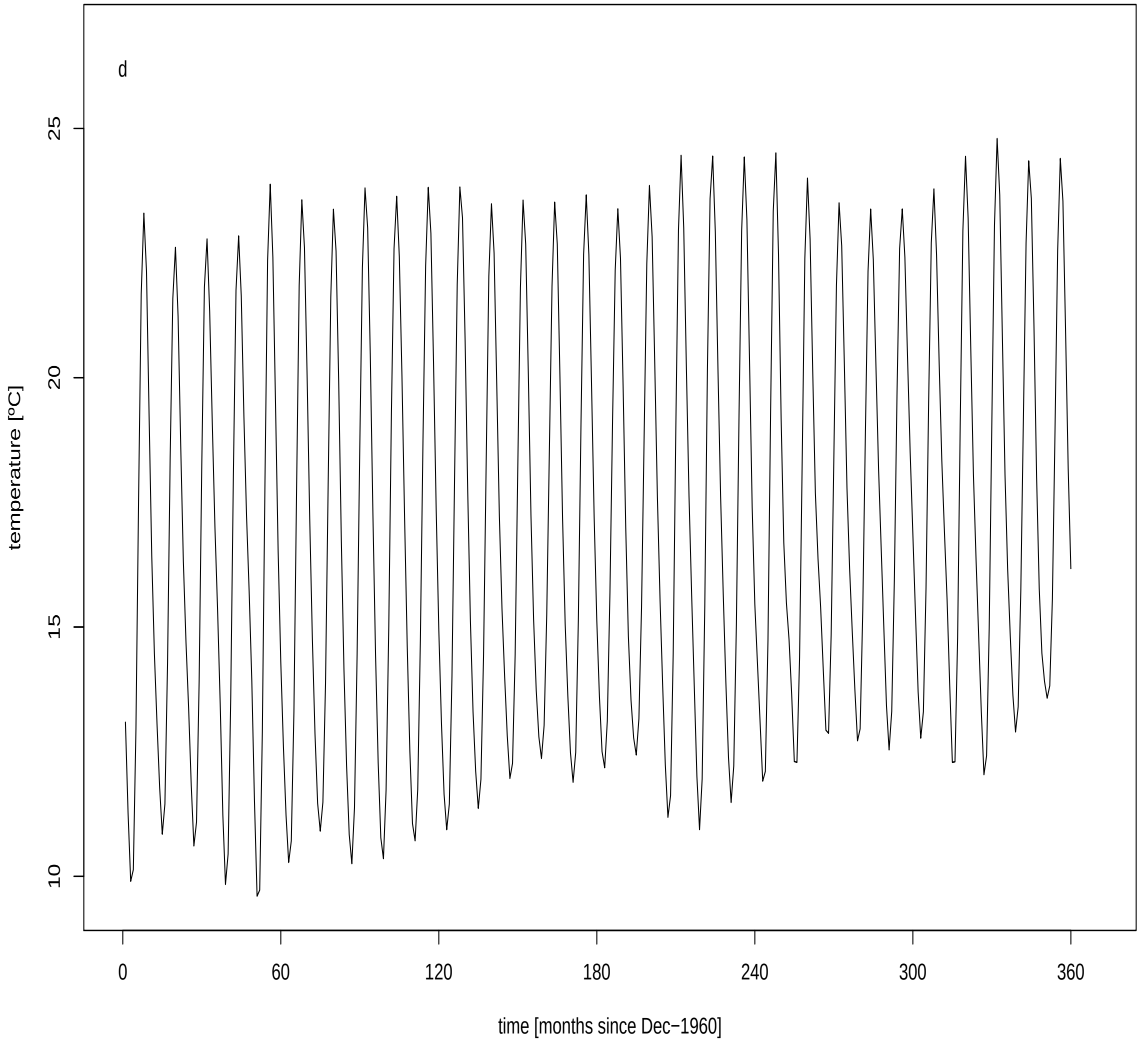
August SST from LS09





Trends (°C/30 years)

Posterior mean for monthly SST at s_2 (°C).



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• The previous two points imply that we need to consider a hierarchical structure for our statistical model that is inspired by physical equations.

We consider observations averaged over grid cells indexed by $\boldsymbol{i} = (i_1, i_2, i_3) \in \mathbb{I}$. The observation equation is given by

$$\begin{pmatrix} T_t(\boldsymbol{i}) \\ S_t(\boldsymbol{i}) \end{pmatrix} \sim N_2 \left(\boldsymbol{\theta}_t(\boldsymbol{i}), \begin{pmatrix} \frac{\tau_T^2}{n_{T,t}(\boldsymbol{i})} & 0 \\ 0 & \frac{\tau_S^2}{n_{S,t}(\boldsymbol{i})} \end{pmatrix} \right)$$

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The process equation is

$$\boldsymbol{\theta}_t(\boldsymbol{i}) \sim N_2\left(\sum\nolimits_{\boldsymbol{j}} K[\boldsymbol{i}-\boldsymbol{j}, \boldsymbol{w}(\boldsymbol{i})]\boldsymbol{\lambda}_t(\boldsymbol{j}), \boldsymbol{C}(\boldsymbol{i})\right)$$

$$\boldsymbol{C}(\boldsymbol{i}) = \tau_{\theta_1}\tau_{\theta_2}\begin{pmatrix} \frac{\tau_{\theta_1}}{\tau_{\theta_2}} & \Phi\left(\sum_{\boldsymbol{j}} K[\boldsymbol{i}-\boldsymbol{j}]\varpi(\boldsymbol{j})\right) \\ \Phi\left(\sum_{\boldsymbol{j}} K[\boldsymbol{i}-\boldsymbol{j}]\varpi(\boldsymbol{j})\right) & \frac{\tau_{\theta_2}}{\tau_{\theta_1}} \end{pmatrix}$$

with $\boldsymbol{j}$ in a grid $\mathbb{J}$, $\boldsymbol{w}$ a DPC and an MRF for ϖ.

The Model for the Latent Process

$$\boldsymbol{\lambda}_t(\boldsymbol{j}) = \boldsymbol{\lambda}_{t-1}(\boldsymbol{j}) + \boldsymbol{\alpha}_{t-1}(\boldsymbol{j})s_t + \boldsymbol{\beta}_{t-1}(\boldsymbol{j})c_t + \boldsymbol{\gamma}_{t-1}(\boldsymbol{j}) + \boldsymbol{\epsilon}_t^{\lambda}(\boldsymbol{j})$$

$$\boldsymbol{\alpha}_t(\boldsymbol{j}) = \boldsymbol{\alpha}_{t-1}(\boldsymbol{j}) + \boldsymbol{\epsilon}_t^{\alpha}(\boldsymbol{j})$$

$$\boldsymbol{\beta}_t(\boldsymbol{j}) = \boldsymbol{\beta}_{t-1}(\boldsymbol{j}) + \boldsymbol{\epsilon}_t^{\beta}(\boldsymbol{j})$$

$$\boldsymbol{\gamma}_t(\boldsymbol{j}) = \boldsymbol{\gamma}_{t-1}(\boldsymbol{j}) + \boldsymbol{\epsilon}_t^{\gamma}(\boldsymbol{j})$$

with $(\boldsymbol{\epsilon}_t^{\lambda}(\boldsymbol{j}), \boldsymbol{\epsilon}_t^{\alpha}(\boldsymbol{j}), \boldsymbol{\epsilon}_t^{\beta}(\boldsymbol{j}), \boldsymbol{\epsilon}_t^{\gamma}(\boldsymbol{j}))' \sim N_8\left(\mathbf{0}, \boldsymbol{W}_t(\boldsymbol{j})\right)$, $\boldsymbol{W}_t(\boldsymbol{j})$ obtained using discount factors, $s_t = \sin\left(2\pi t/52\right)$ and $c_t = \cos\left(2\pi t/52\right)$.

The Model for Velocities

The physical constraints are imposed by assuming dependence on the zonal and meridional velocities, $\upsilon_t(\boldsymbol{i})$ respectively, and $\nu_t(\boldsymbol{i})$ and the density $\rho_t(\boldsymbol{i})$.

$$\upsilon_t(\boldsymbol{i}) \sim N^{(\upsilon_{\min}(\boldsymbol{i}),\upsilon_{max}(\boldsymbol{i}))}\left(m_\upsilon, \tau_\upsilon^2\right)$$

$$\nu_t(\boldsymbol{i}) \sim N^{(\nu_{\min}(\boldsymbol{i}),\nu_{max}(\boldsymbol{i}))}\left(m_\nu, \tau_\nu^2\right)$$

where, assuming geostrophic conditions,

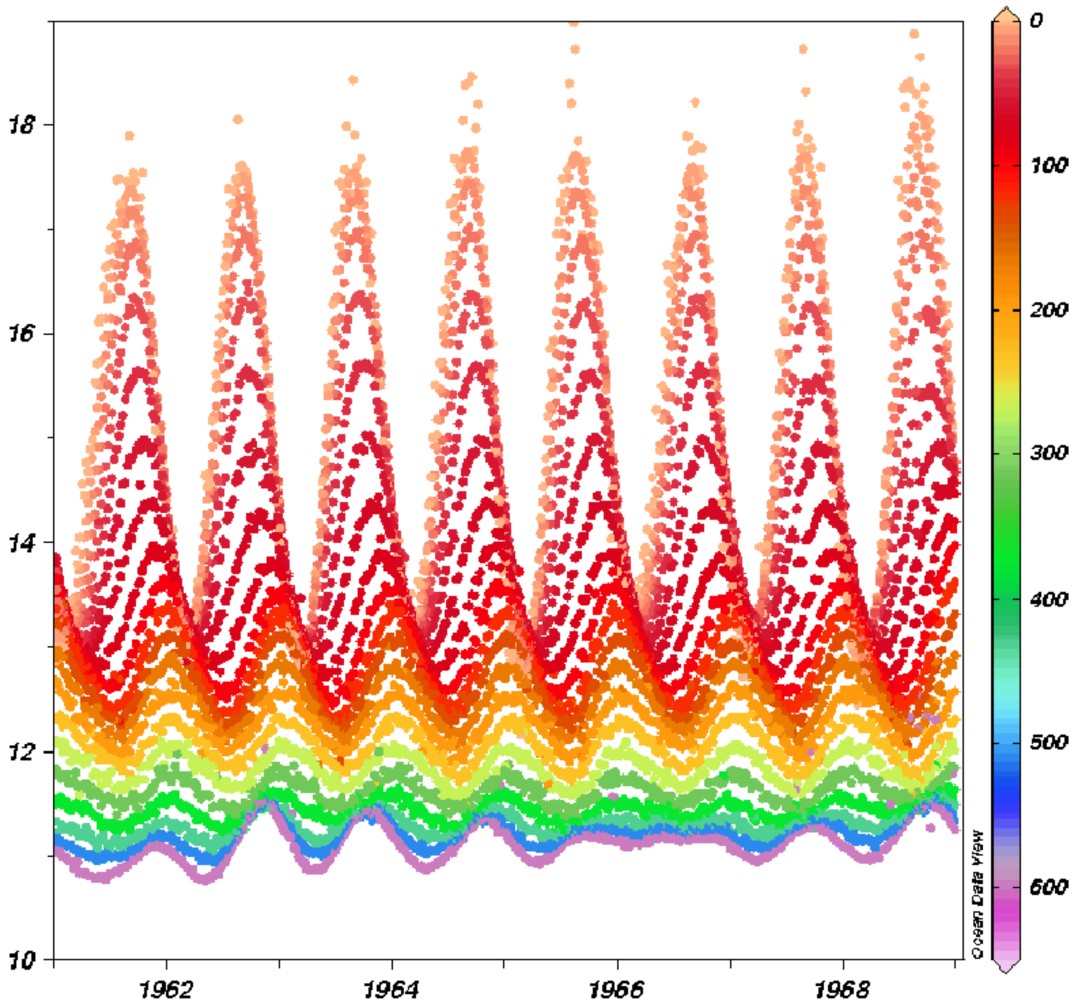
$$m_\upsilon = \frac{z(\boldsymbol{i})g(\boldsymbol{i})}{2\rho_t(\boldsymbol{i})f_c(\boldsymbol{i})}\frac{\partial\rho_t(\boldsymbol{i})}{\partial y} \quad \text{and} \quad m_\nu = -\frac{z(\boldsymbol{i})g(\boldsymbol{i})}{2\rho_t(\boldsymbol{i})f_c(\boldsymbol{i})}\frac{\partial\rho_t(\boldsymbol{i})}{\partial x}$$

Here $z(\boldsymbol{i})$ is thickness, $g(\boldsymbol{i})$ is the gravity acceleration and $f_c(\boldsymbol{i})$ is the Coriolis parameter.

Density is determined by the equation of state, $f_w(\cdot)$ that provides density as a function of temperature, salinity and pressure (depth). Thus

$$\rho_t(\boldsymbol{i}) \sim N^{(\rho_t(\mathcal{A}(\boldsymbol{i})),\rho_t(\mathcal{B}(\boldsymbol{i})))}\left(f_w\left(\boldsymbol{\theta}_t(\boldsymbol{i}), p(\boldsymbol{i})\right), \tau_\rho^2\right)$$

$\mathcal{A}(\boldsymbol{i})$ and $\mathcal{B}(\boldsymbol{i})$ denote respectively the grid points of $\mathbb{I}$ immediately above and below $\boldsymbol{i}$.



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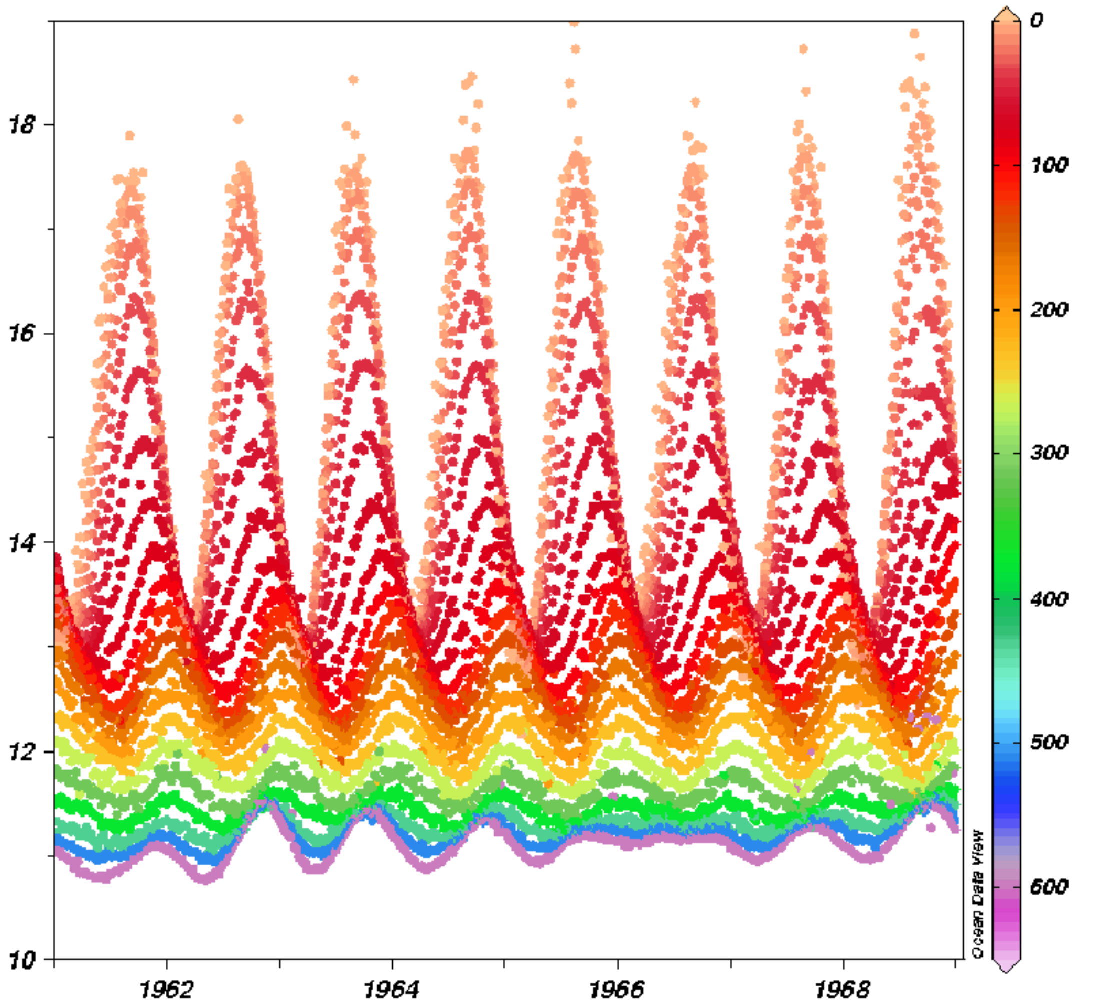
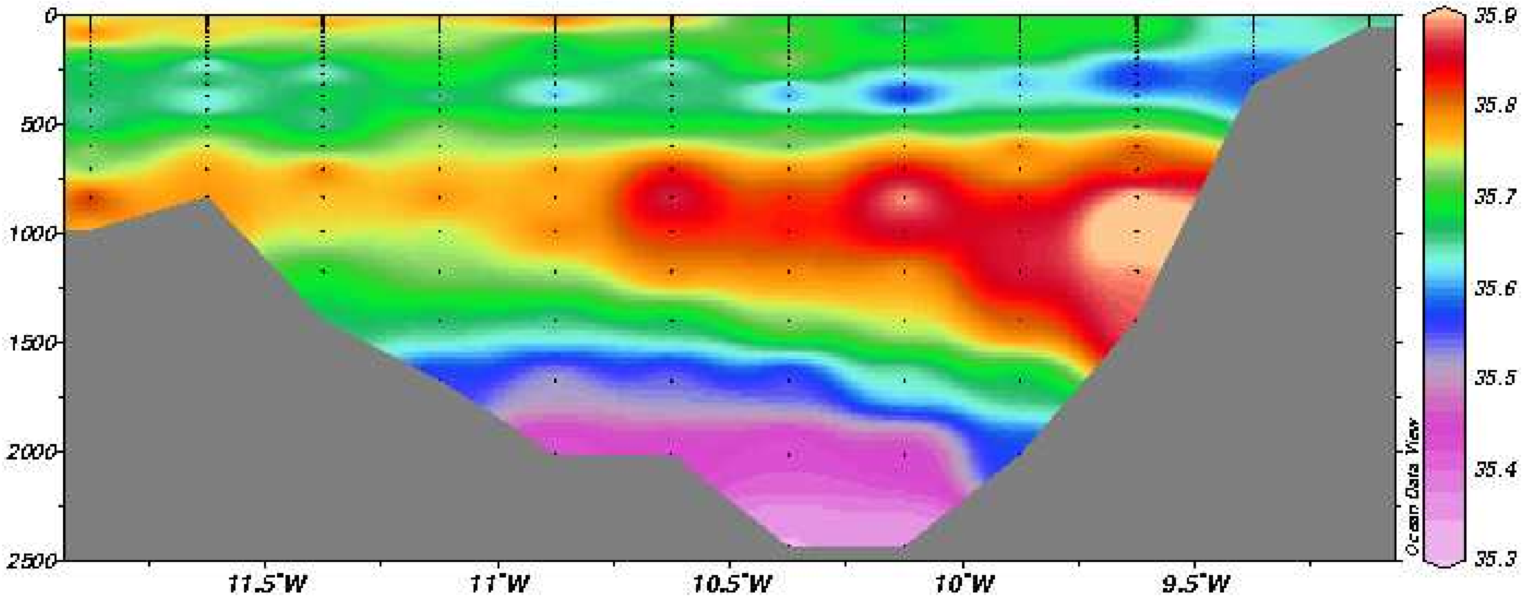
Temperature Variability

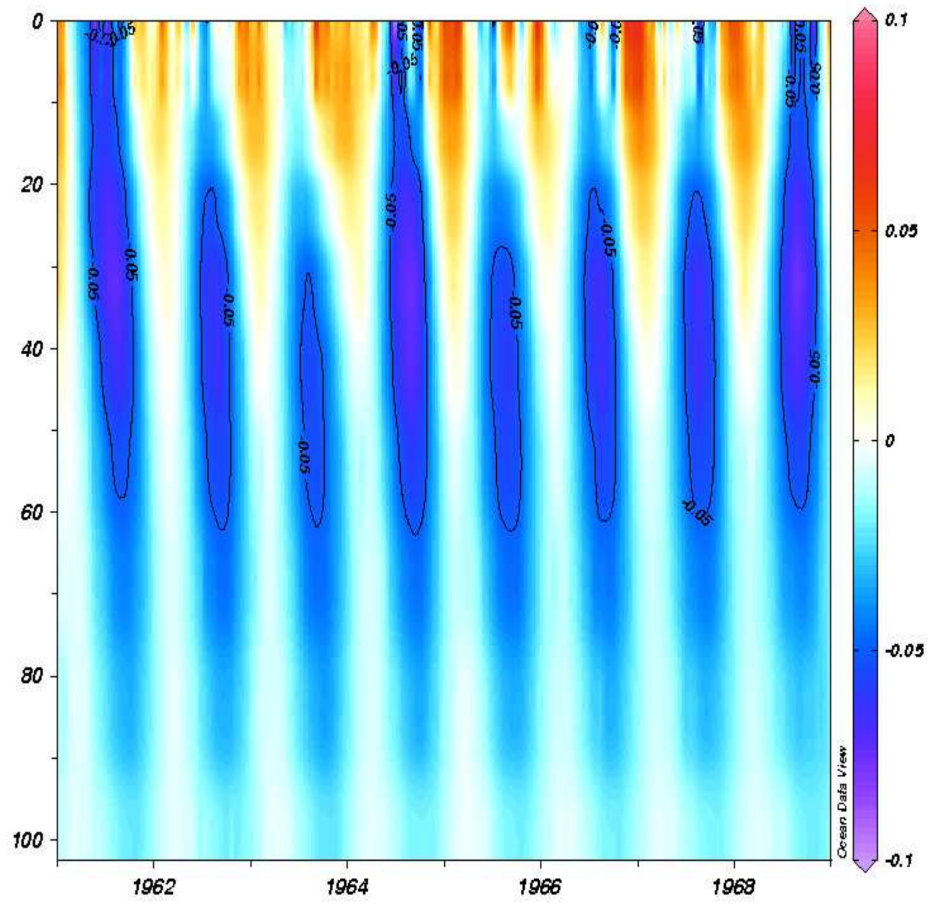


- We run the 3D joint model for weekly data corresponding to the Iberian Peninsula from 1961 to 1970.
- The figure shows the estimated temperatures for different depths in an area off the coast of Galicia.



Estimated salinity field for an area off the coast of Galicia in July 1961. We observe a very clear plume of salty water close to the coast that is known to come from the Mediterranean Sea.

Time varying thermocline at Cape St. Vincent.



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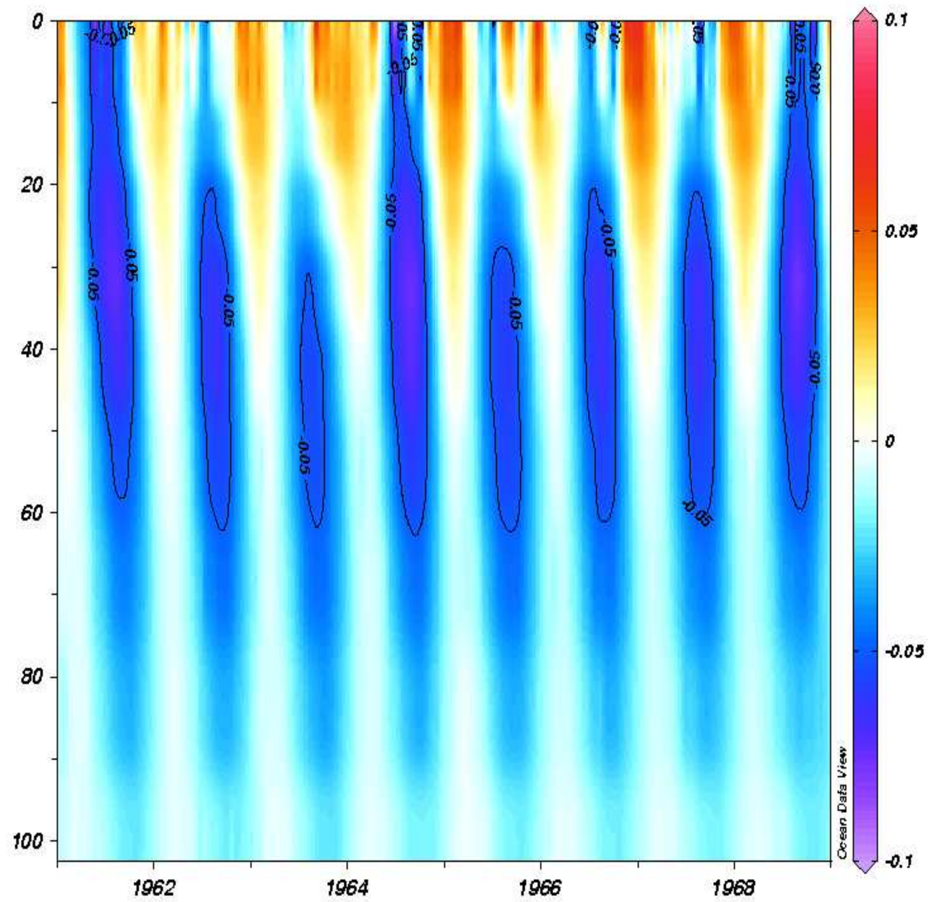

- The Thermocline is a thin ocean layer where the temperature gradient is high relative to the upper and lower layers.
- It separates, in terms of temperature, the high varying surface waters to the smoothly varying deep ocean waters.

Discussion

- We have developed spatio-temporal models for ocean variables that are non-stationary in either space or time. We are able to capture important space- and time-varying features of the ocean.

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- Our model includes observational errors and realistic descriptions of the latent processes governing the evolution of ocean variables.
- The model is able to provide probabilistic assessments of the variabilities included in the estimated quantities. All estimation variabilities are accounted for in the final product.

- Our model is able to handle large data sets. By using kernels with compact support and making use of the structure of the CDLM we are able to parallelize the estimation algorithms.

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