Navigating Blindly

Multidimensional online robot motion in unknown environments

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22 October, 2008

Outline

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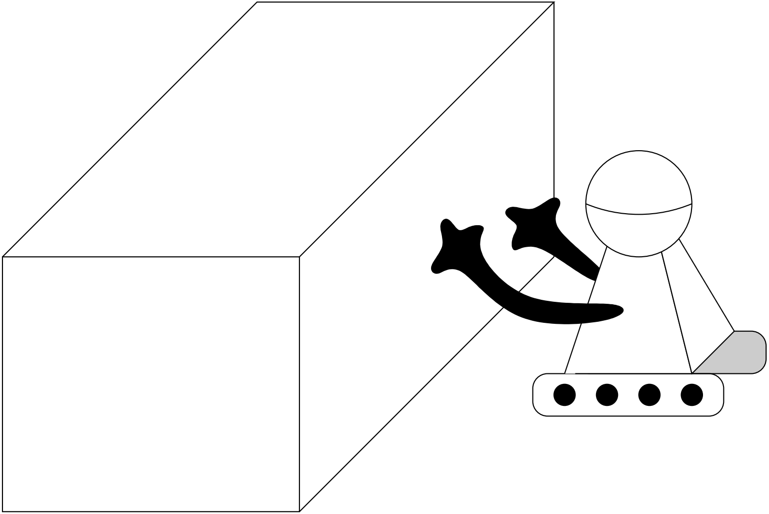

Our Three Tasks

- Given:
  - An environment $X \subset \mathbb{R}^n$ with finite diameter
  - A spherical robot, named Bob, of radius $r > 0$, equipped with:
    - tactile sensor
    - GPS sensor
  - A starting point $S \in X$
  - Possibly, a target point $T \in X$
- The tasks are:
  - $COVER_n$: Occupy as much of X as possible
  - $SEARCH_n$: Find T and move from S to T
  - NAV_n: Move from S to T
- We want an *efficient* algorithm solving the task.
  - offline if X is known
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- navigation problems
  - mail delivery in a city
  - moving packages in a factory
- configuration space problems
  - Shuttle arm motion
- exploration and sample acquisition
  - Mars Rover
- area coverage problems
  - cleaning public places
  - Roomba (video)

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Mathematical Motivation

- Discrete approximation via Rips complexes

Theorem (Caraballo)

Let C be a compact subset of $\mathbb{R}^n$. For any point $q \in \mathbb{R}^n$ and for almost every $r > 0$:

$$Vol_{n-1}((d_C^{-1}(r)) \cap B^n(q, 2r)) \leq 4^{n+1} r^{n-1}.$$

- Even if C is a fractal curve, most tubes about C have finite surface area.
- Let C be the set of points where Bob's center can be. Almost always, the boundary of C has finite volume.

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The BUG1 algorithm

[Lumelsky and Stepanov]

BUG1

While not at T:

- Move directly towards T.
- **If** an obstacle is encountered:
  - Explore the obstacle (via clockwise circumnavigation).
  - *Move* to some point p_{min} on the obstacle closest to T.
  - **If** Bob cannot move directly towards T from p_{min}:
    - Target unreachable.

Target reached

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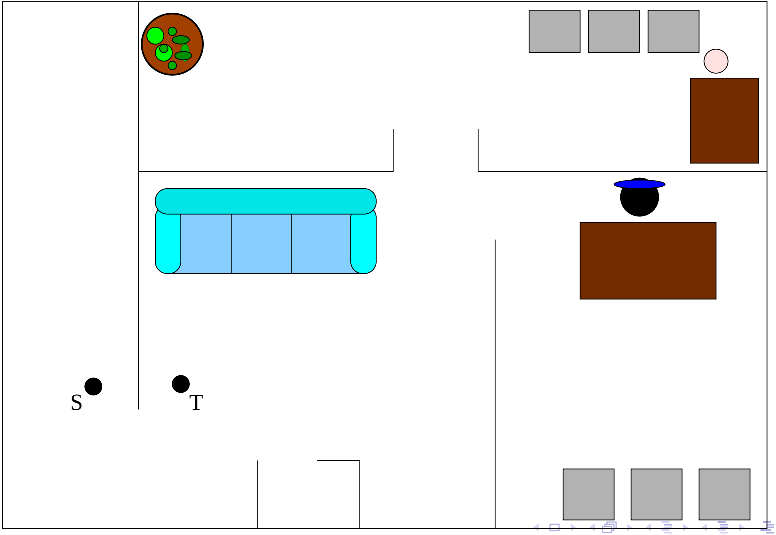

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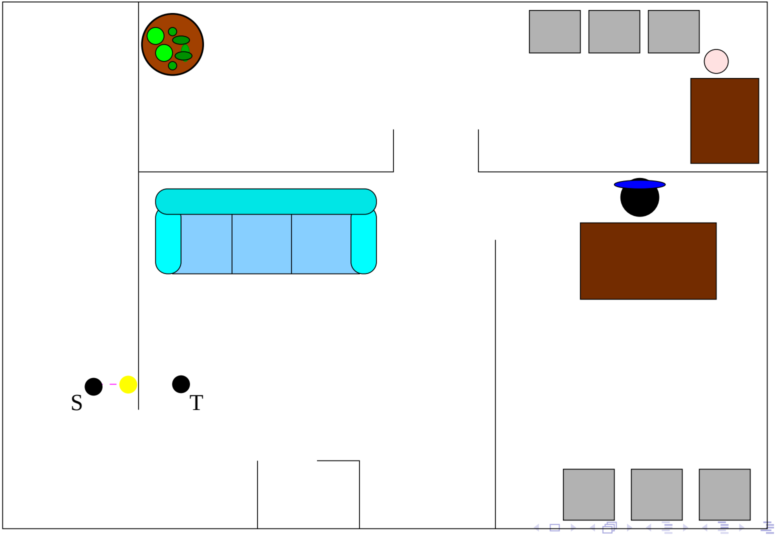

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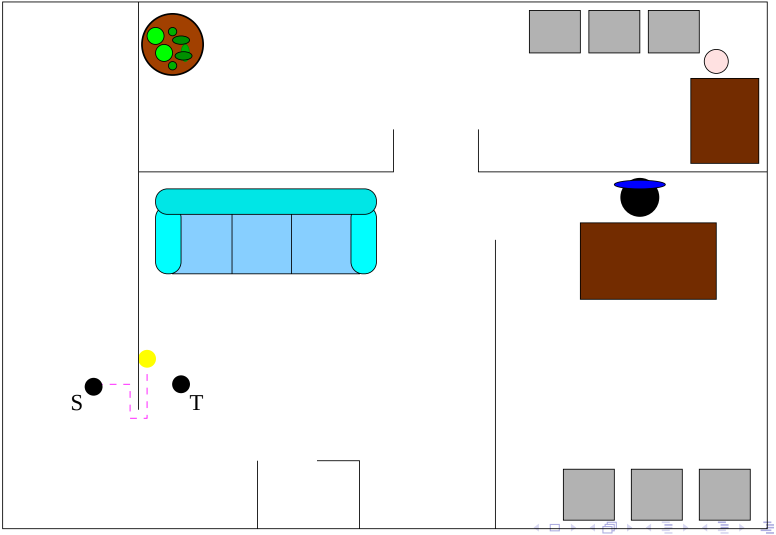

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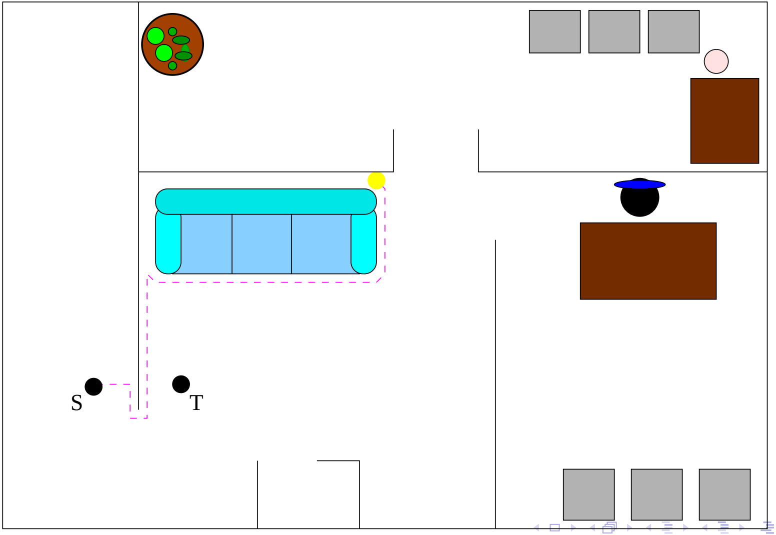

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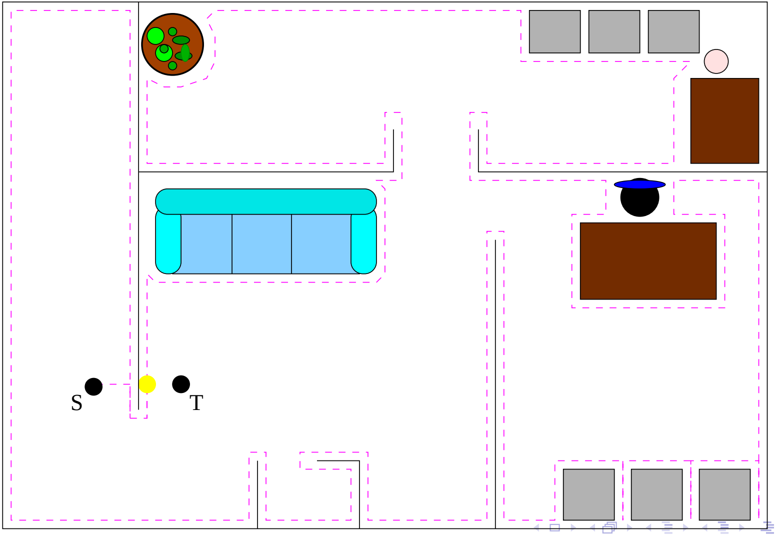

Definition of Competitiveness

- Given task P, like NAV_n
- For any algorithm A solving P, define

$$f_A(t) = \sup\{t_A(X) | t_{opt}(X) \leq t\}$$

- $g : \mathbb{R} \to \mathbb{R}$ is a *universal lower bound* on competitiveness of P if for all A,

$$f_A \in \Omega(g)$$

- A is $O(g)$*-competitive* if

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- A is *optimally competitive* if A is $O(g)$-competitive and g is a ULB for P

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Competitiveness

- Competitiveness can be linear, quadratic, exponential, etc.
- A is linearly competitive iff $f_A(t) \in O(t)$ iff there exists c_1, c_0 with $t_A(t) \leq c_1 t_{opt} + c_0$
- Example: Tree traversal.
  - Goal: visit each vertex of a tree and return to start.
  - Algorithm: Never go back across an edge until all neighboring edges have been traversed twice.

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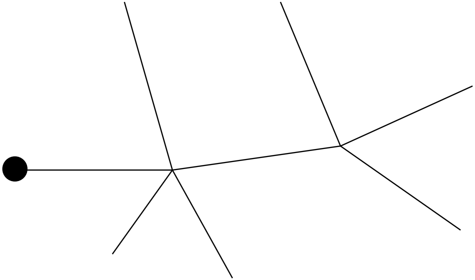

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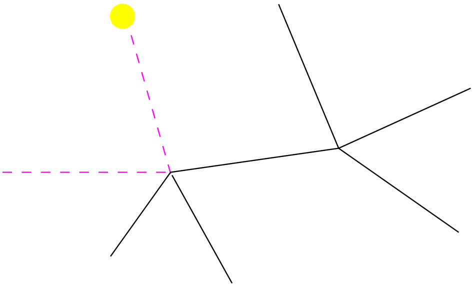

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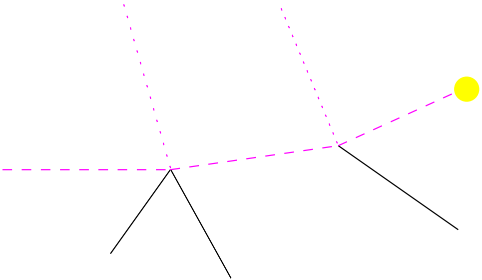

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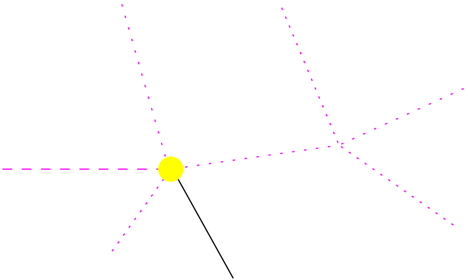

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  - Must traverse each edge twice, and algorithm traverses each edge exactly twice, so optimally (linearly) competitive.

Competitiveness of BUG1

Competitiveness of BUG1?

- Horrible.
- Runs in time proportional to sum of lengths of boundaries of (intervening) obstacles.
- Is not $O(g)$-competitive for any function g.

How to fix it? Don't allow Bob to get too far from T. Introduce ellipse as virtual obstacle.

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CBUG

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Fix initial area $A_0(\sim d(S,T)^2)$

For $i = 0$ to ∞:

- **Execute** BUG1(S,T) within ellipse with foci S and T and area $2^i A_0$.
- Success if at T
- Failure if Bob did not touch virtual ellipse

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[Gabriely and Rimon]

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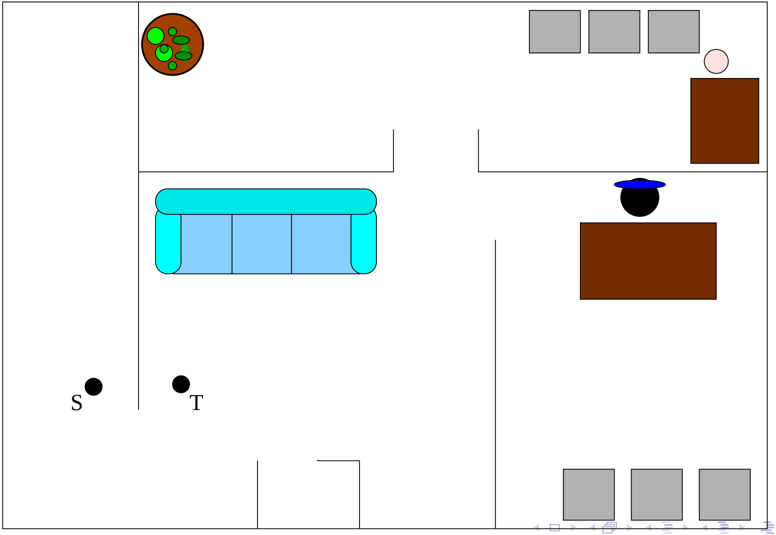

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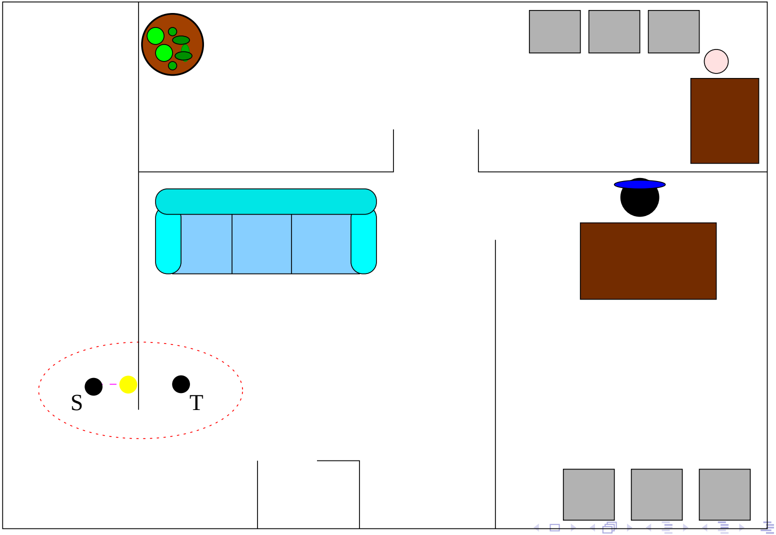

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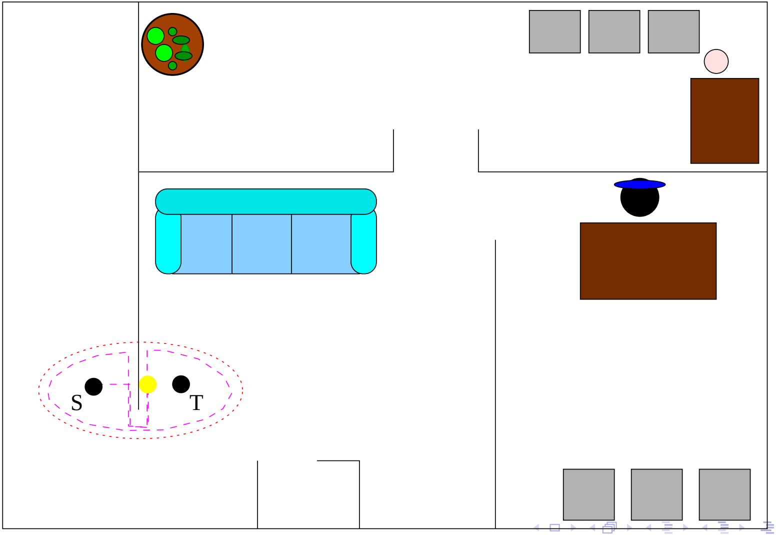

Competitiveness of CBUG

(Show Program)

Geometric Progression.

Theorem (Gabriely and Rimon)

NAV_2 has a quadratic universal lower bound, namely given by

$$g_r(x) := \frac{2\pi}{3(1+\pi)^2 r} x^2 \sim \frac{.122x^2}{r}.$$

Theorem (Gabriely and Rimon)

If T is reachable, CBUG solves NAV_2 in time at most

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- Proof of second theorem has a flaw....
  - Let $A =$ *area covered*. Gabriely and Rimon assume length of path traversed is at most $\frac{A}{2r}$.
  - Analogue not true for higher dimensions (ex: 3-D r-neighborhood of planar space-filling curve)
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If $n \geq 3$, then every algorithm that solves either NAV_n or $SEARCH_n$ is not $O(f)$-competitive for any $f : \mathbb{R} \to \mathbb{R}$.

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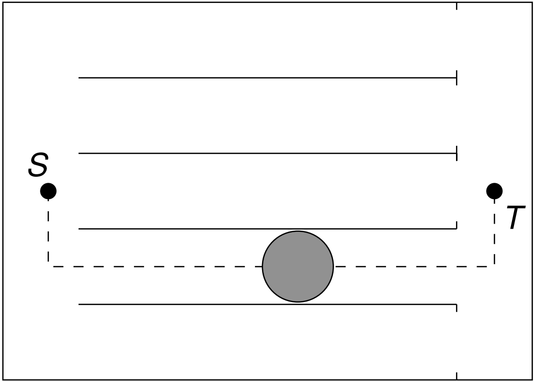

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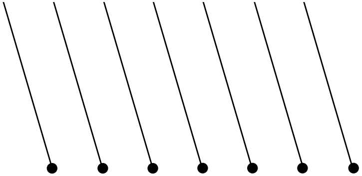

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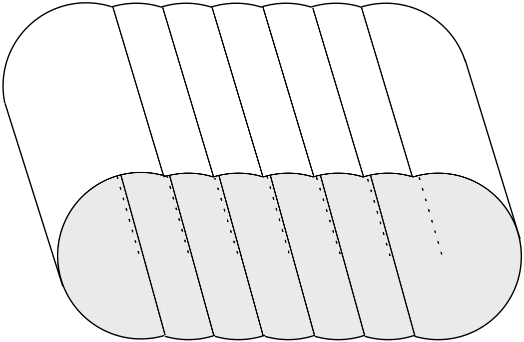

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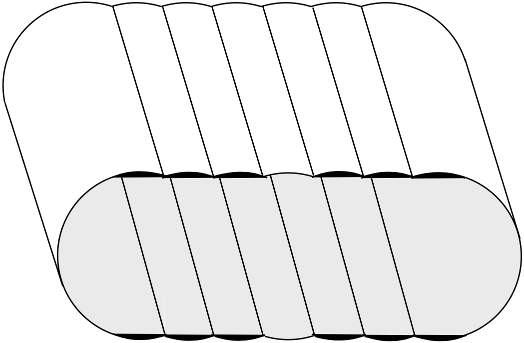

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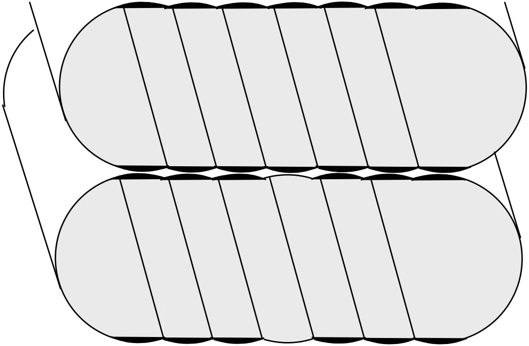

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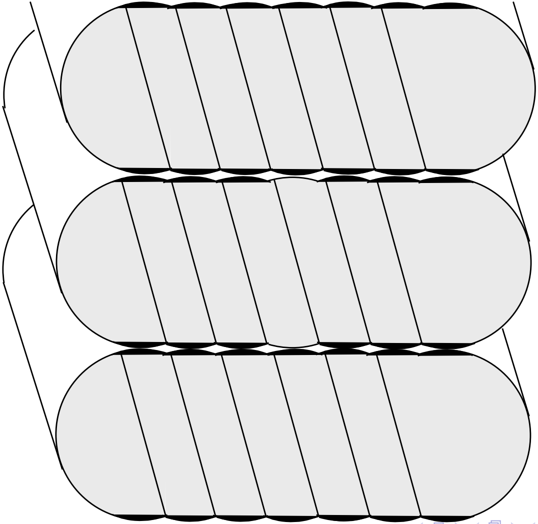

The Fix: Clearance Parameter

- Weaken tasks slightly by adding a *clearance parameter*, $\epsilon > 0$.
- Introduction of ϵ allows us, for instance, to ignore parallel corridor spaces where the corridors are packed too tightly.
- For a fixed ϵ, define

$$\kappa = 2\sqrt{2r\epsilon + \epsilon^2}$$

and

$$r' = r + \epsilon.$$

- If a robot of radius r' can occupy two points A and B of X, and $d(A, B) < \kappa$, then Bob can move freely along the straight line from A to B.

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The *modified NAV_n and $SEARCH_n$* problems are to reach T if there is an r'-path from S to T, and otherwise reach T or determine no r'-path exists.

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The *modified $COVER_n$* problem is to come within r' of every point within r of an r'-path from S.

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Linear universal lower bound for $COVER_n$.

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- Proof: Analyze parallel corridor spaces (lots of details to check)
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- Cannot explore obstacles by touching every point
- Remember explored obstacle points
- Key observations: Only need sufficiently fine mesh of obstacle points: 2 obstacle points of distance less than $2r$ apart prevent Bob from passing between them
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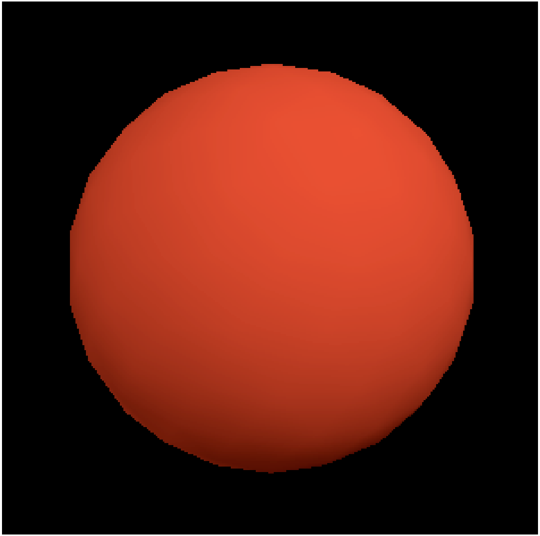

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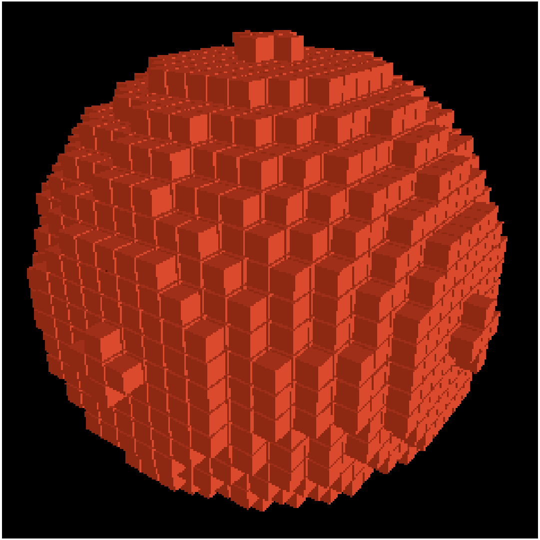

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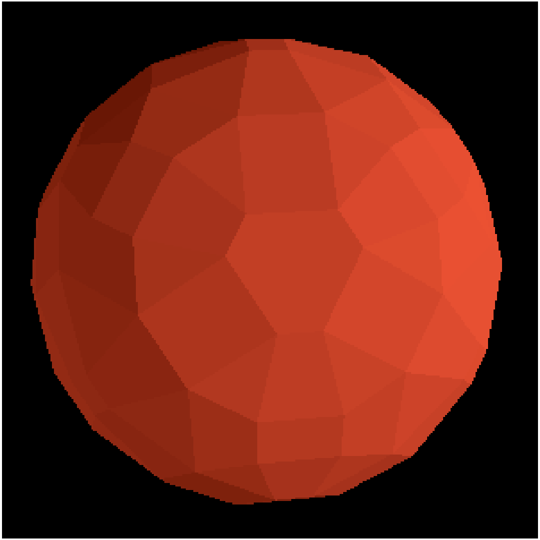

Colors

- White: Unexplored;
- Yellow: Bob's center can be at the center of the cube;
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Boxes$_\epsilon$

- Break X into a grid of axis-parallel cubes of side length $l = \min\{\epsilon/2, \epsilon/\sqrt{n}\}$. Color all cubes White.
- Move from S to the center of the current cube, C. Stop if an obstacle is found.
- Define $a_0 = d(S, T') + l$, and set $a = a_0$.
- **While** not in the same cube as T
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  - Explore X using GraphTraverse(C).
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  - Double a and color all Pink cubes White.
- Travel towards T. If an obstacle is encountered, stop.

GraphTraverse

GraphTraverse(C)

- **If** $T \in C$ Return.
- Set $Adjacent = \{$cubes sharing $(n-1)$-face with $C\}$.
- **While** there are White cubes in $Adjacent$,
  - Pick White $D \in Adjacent$.
  - Move in a straight line toward the center of D.
  - **If** obstacle is hit, color D red and return to center of C.
  - **Else**
    - Color D Yellow
    - GraphTraverse(D,T).
    - **If** T is in the current cube, Return.
  - Travel back to the center of C.
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Analysis of Boxes

Theorem (BKS)

If there is an $(r + \epsilon)$-path, p, from S to T, then $Boxes_\epsilon$ will move Bob from S to T.

Sketch of proof:

- points in a cube are at most ϵ apart
- If the center of a robot of radius r' can be SOMEWHERE in a cube, Bob can be ANYWHERE
- Reduces problem to finite graph exploration: stick to 1-skeleton of dual
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In spaces without bottlenecks, the length of the path generated by $Boxes_\epsilon$ is at most $c_n(l_{opt})^n\left(\frac{1}{\epsilon}\right)^{n-1} + \frac{d_n}{\epsilon} + \epsilon$ for some constants c_n, d_n depending only on n.

- Sketch of Proof: tree traversal
- Compare to universal lower bound of

$$\frac{l_{opt}^n}{\kappa^{n-2}r'} \sim \frac{l_{opt}^n}{\epsilon^{\frac{n-2}{2}}r'}$$

for small ϵ (recall $\kappa = 2\sqrt{2r\epsilon + \epsilon^2}$).

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Solving *COVER*

Theorem (BKS)

The algorithm CBoxes solves the modified $COVER_n$ problem and is optimally competitive with an upper bound on competitiveness given by $cl_{opt} + d$, where c and d are constants depending on r, n, and ϵ.

Improvements

Our algorithms are bare-bones to prove complexity estimates. Average-case runtimes greatly improve with some modifications:

- Sampling Improvement by using better lattice
- Taking Diagonals Improvement
- Noticing T Improvement to treat *SEARCH* like *NAV*
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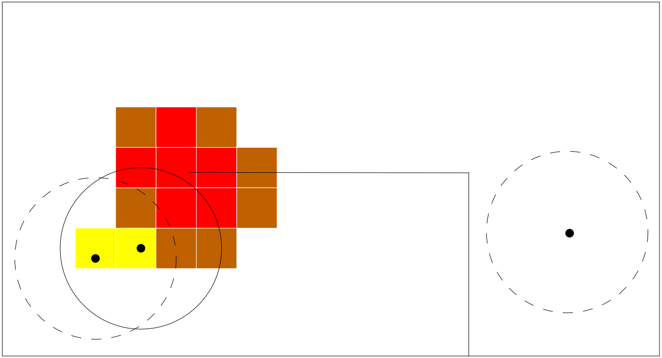
Example of Boxes (improved)



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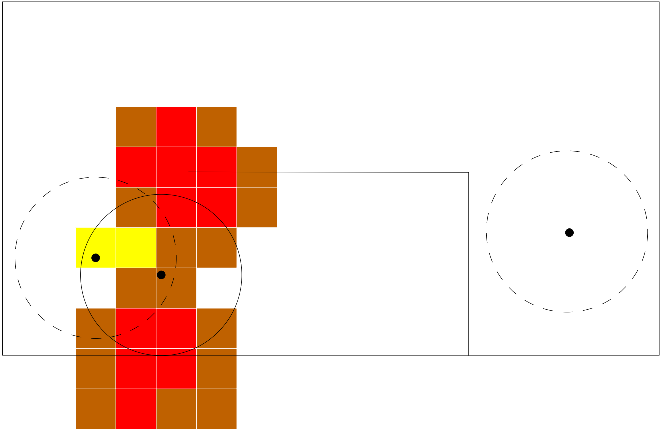

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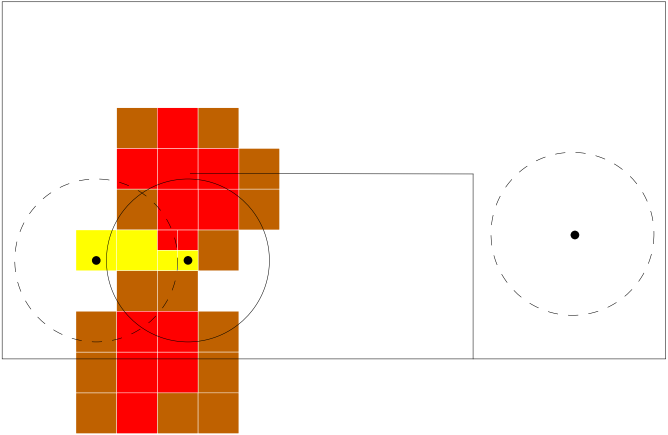

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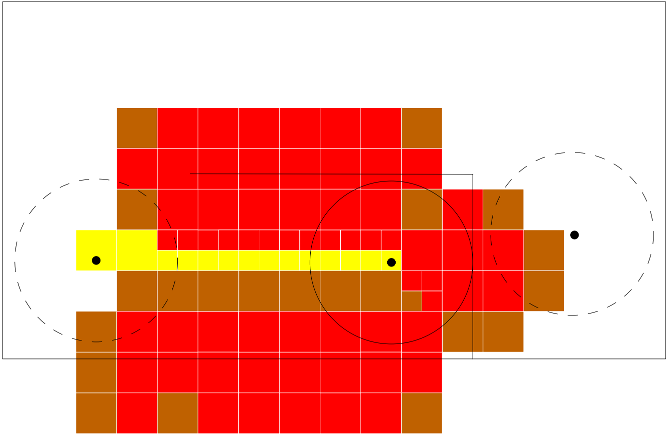

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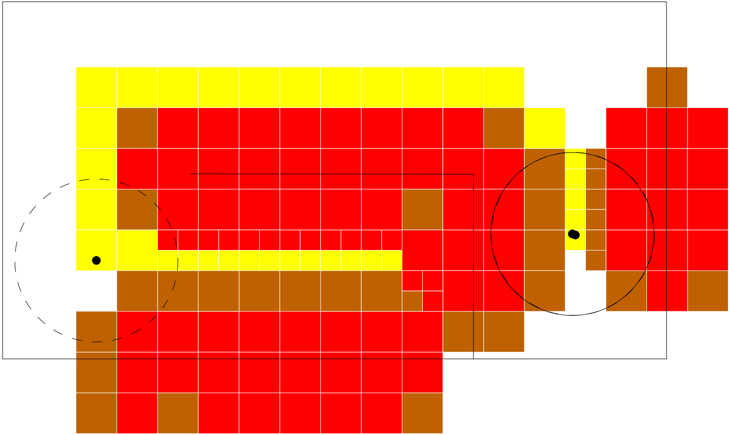

Future Directions

- Improve Boxes to be optimally competitive.
- Calculate average-case complexity with improvements in certain environments
- Analyze case if T is unreachable – 'discompetitive analysis'.
- Program implementation.
- Non-spherical robots, other coordinate systems (tori).
- Consider various applications (arm linkages, Roomba).
- Apply to coordinate-free search and exploration problems (Mars rover).

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